Algebra's Tragic Hero

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Abstract

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Évariste Galois had created beautiful mathematics before he died in a duel on May 31, 1832 at the age of 20. In his lifetime, even the most esteemed mathematicians of France's *Académie des Sciences* could not understand his work, and only decades after his death did mathematicians realize the true beauty within Galois's work. Today it forms a pillar of abstract algebra called Galois Theory. Given the novelty of his work and its obscurity, Galois did not receive as much acclaim as he would have liked or as his theory deserved. Thus, over the years after his death, the Galois legend became one of a misunderstood genius persecuted and driven away from mathematics by the old guard at the *Académie*. Naturally, the circumstances around his death, never explicitly stated, became a topic of some debate as well. What caused Galois's fateful duel, which robbed mathematics of a brilliant mind?

In this paper I offer my own version of the Galois story. After a summary of his life, I reject the relatively recent claim by historian Laura Toti Rigatelli that Galois willfully died in a duel in order to spark a political uprising. Based on, among other evidence, letters that Galois himself wrote the night before he died, I argue instead that he died in a duel over a woman. Next, I provide an introduction to the beautiful Galois theory, which originated from Galois's work. Finally, I discuss one major thread of the Galois legend: that the mathematical community wrongly and consistently rejected him. I trace this claim from the famous 1937 account of Galois's life by writer Eric Temple Bell back to Galois himself. Documents from two prominent *Académie* mathematicians, Augustin-Louis Cauchy and Siméon Denis Poisson suggest the contrary: the *Académie*, though it rejected Galois's work, had much more faith in Galois's work than Galois believed. The tragedy of Galois's life is not that he went unrecognized until years after his death. It was that he lost his life in a duel when he had been on a trajectory to climb the ranks of the mathematical community that he had believed had rejected him.

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1 Introduction

Évariste Galois was a legendary French mathematician who lived from 1811 to 1832, dying only when he was twenty years old. He was the archetypical brooding genius. As one of his teachers noted, "the furor of mathematics possesses him" [9, 208]. Yet, the legend goes, the mathematical community unjustly ignored his innovative work, as Galois was constantly rejected by the ossified scientific establishment at the *École Polytechnique* and the *Académie des Sciences*. At the same time, Galois was a political firebrand and was twice arrested. After imprisonment and a stunted love affair, Galois found himself condemned to a pistol duel, presumably over the honor of the woman with whom he had had a disappointing courtship. Galois supposedly knew that he would perish in the duel. So, on the night before, as Eric Temple Bell famously wrote, he "spent the fleeting hours feverishly dashing off his scientific last will and testament" [2, 375] to send to a close friend, which would change the course of mathematics. His opponent mortally wounded him, and Galois died the following morning at the age of 20, seeming to fulfill his prophecy he had made to a friend while drunk in prison only months before: that he would die over a "*coquette de bas étage*."

More than a decade after his death, mathematicians began to appreciate Galois's work, and over the years they and historians of mathematics developed the Galois legend, which remains today as an inspiration to young mathematicians and as a warning against rigid scientific institutions. Galois lived on the cusp between the Enlightenment and the Romantic era and during a transition from one guiding view of mathematics to another. Before Galois was born, during the Enlightenment, mathematics was a tool used to answer questions about the natural world, whose methods' worth depended on their practicality. Galois and other mathematicians of his era went against this trend, building instead a mathematics of abstract realms seemingly divorced from reality. Galois's work, though mostly ignored during his life, led to the birth of modern abstract algebra decades after his death and merits him a spot among other mathematical geniuses such as Cauchy and Gauss.

As one might expect with such a legendary figure, many details about his life which mathematicians recount with admiration or anger are in fact not true, owing to the accounts of, among others, Bell himself. For example, though Galois's work failed to earn the approval of the prestigious *Académie*, he nevertheless published some of his papers in several journals. At least part of his failure to find acceptance among the larger scientific community was due to his hot temper, his propensity for obscurity in his mathematical arguments, and his radical political views. He got himself expelled from school by criticizing its director's politics. As for the duel, writers have offered several competing theories. Often, they argue that the duel was politically motivated: either it was a set-up by the police or a deliberate martyrdom by Galois's own will in order to spark a political uprising.

These theories are often contrived and ignore countervailing evidence. Three letters written by Galois, two to his friends and one addressed to his fellow Republicans, offer enough evidence to disprove most of these theories. A few days before the duel, for example, he wrote to his close friend Auguste Chevalier that he would visit him in a couple weeks and planned to move to another part of France. In the letter to his compatriots, written the night before the duel, he asked that they "not reproach [him] for dying other than for [his] country" and lamented that instead he would die in a frivolous quarrel over "a *coquette* and her two dupes." Galois's own words offer evidence for the cause of his fatal duel: love.

Another embellished thread of the Galois legend is that he was treated unfairly by other mathematicians. Though Galois did struggle to publish his papers, his case is not unique in that regard from other young mathematicians at the time (or at any other time). Two members of the *Académie*, Cauchy (who, contrary to legend, did not simply lose Galois's work) and Poisson, even encouraged Galois to continue his work. Several mathematicians have offered commentary on the work Galois submitted and agreed that his proofs were concise to a fault. Ultimately, it was Galois's lack of time that prevented him from gaining in life the recognition he deserved.

What has not been fabricated or embellished, however, is Galois's influence on mathematics and on mathematicians. In 1846, fourteen years after Galois's death, the mathematician Joseph Liouville published Galois's mathematical papers, many of them for the first time. Although Galois lacked clarity in his arguments, his beautiful theory helped usher in a new era of mathematics. His work has earned him the title of "genius."

2 Sources

I use two sources for Galois's writings: *Galois, Écrits et Mémoires* (*Galois, Writings and Memoirs*), by Robert Bourgne and J.-P. Azra (1962), and *The Mathematical Writings of Évariste Galois*, by the British mathematician Peter M. Neumann (2011). The former contains all of Galois's known writings, including his letters and notes he wrote in the margins of his manuscripts. The latter is an English translation of all of Galois's mathematical writings, and I have referred to it when reading Galois's mathematical papers. On May 29, 1832, the night before Galois died, he wrote a letter to his close friend, Auguste Chevalier. In this letter, which eventually became known as Galois's "testamentary letter," he summarized the mathematical findings that he had made over the past year and tasked Chevalier with printing the letter in the magazine *Revue Encyclopédique*. Chevalier and Galois's mother collected and copied whatever written work Galois had in his room after his death, including Galois's last letters, and we owe it to them that much of Galois's work has survived. With the help of Galois's brother, Alfred, Chevalier convinced the mathematician Joseph Liouville to publish some of Galois's mathematical manuscripts in his *Journal de Mathématiques pures et appliquées* in 1846, which served as Galois's "first resurrection" [5, IX].

In addition to Galois's own writings, I have used many newspaper and magazine articles pertinent to Galois's life. Some of these were written during his lifetime, but the majority were written afterwards. Many mathematicians, historians, and writers have offered commentary on Galois's life and works. One of the most notable of these is the mathematician Eric Temple Bell. His 1937 book, *Men of Mathematics*, contained a brief, sensational chapter on Galois's life and is the most famous example of the Galois legend. The other major source is Paul Dupuy, who in 1896 wrote the most comprehensive biography of Galois. Dupuy's work informs much of my section on Galois's life. As for Galois's time in prison, the only primary source we have is François-Vincent Raspail, a famous biologist and political radical, who was in prison with Galois. He documented his time in prison in his memoir *Lettres sur les prisons de Paris* in 1839. Another biographer of Galois is Laura Toti Rigatelli, who wrote her biography a century after Dupuy in 1996. Although her work is comprehensive, I argue that she fabricated the circumstances surrounding Galois's duel in her account. I have not endeavored to comb through the rest of her book for such errors, but I have avoided citing her for factual information.

Lastly, I have drawn on my own mathematical knowledge for much of the chapter on Galois's mathematics. I examine primarily the work of his *Premier Mémoire*, in which he proves his famous necessary and sufficient condition for a polynomial to be solvable by radicals. In the almost two centuries since his death, the presentation of the theory has changed significantly from the form in which he gave it. He did not have, for example, the aid of set theory (to be developed by Georg Cantor in 1874) to define a group or a field, whereas I use these definitions to explain the concepts. In that chapter I point out some of the differences between the modern view of Galois Theory and Galois's own notion of it. Ultimately my aim in writing this section is to enable a motivated non-mathematician to better appreciate Galois's obscure yet beautiful ideas.

3 Galois's life

3.1 Brief historical and political context

Galois believed fervently in republicanism. A core tenent of this philosophy is that power in government ought to belong to the governed. In Galois's time, several radical republican organizations resisted the imposition of monarchical rule. Galois and his fellow republicans looked back fondly on the period of the French Revolution from 1789 to 1799. In this time, revolutionaries overthrew (and eventually executed) Louis XVI. Power shifted sharply away from the Catholic Church as well. This period also saw intense political turmoil, including the Reign of Terror, in which Maximilien Robespierre, an official of the newly established republic, and his followers executed thousands of people accused of counter-revolutionary actions. Lynn Hunt's *Politics, Culture, and Class in the French Revolution* gives further details.

A *coup d'état* overthrew the government in 1799 and Napoléon Bonaparte became emperor of the French Empire. Napoléon's fall in 1814 was followed by the Bourbon Restoration, which saw the imposition of a constitutional monarchy until the July Revolution of 1830. This Revolution was a reaction against King Charles X's July Ordinances, which among other things, suspended the freedom of the press, dissolved the elected Chamber of Deputies of France, and disenfranchised the commercial middle class. The backlash in the July Revolution resulted in his exile and the installation of a new king, Louis-Philippe. With Louis-Philippe's coronation came a new constitution which served as a compromise between republicans and those who hoped to preserve the monarchy. For more information, see François Furet's *Revolutionary France: 1770-1880.*

Various radical republican organizations formed during the time of the July Revolution, and many were dissatisfied with the compromises struck in its aftermath. They also believed that Louis-Philippe would renege on the concessions made to republicans in the new constitution. Galois joined one such organization, the *Société des Amis du Peuple* (Society of Friends of the People) and was embroiled in political turmoil during the last two years of his life.

3.2 Galois's first eighteen years

In this chapter I discuss Galois's short life, focusing in particular on three aspects: his political views, his place in the mathematical world during his life, and his opinions on mathematics. Although Galois is remembered today mainly for his mathematics, he also cared deeply about politics. His radical political views got him expelled from the *École Normale* and led to both of his arrests. After examining his life in the context of French politics, we turn to Galois's fraught relationship with the mathematical establishment. Galois would not receive proper honor for his work until Joseph Liouville published it years after the young mathematician's death in 1832.

With the exposition in this chapter on Galois's life as a basis, I will critique some of the myths surrounding him, promulgated in large part by the mathematician and writer Eric Temple Bell in 1937. Galois does not fit the archetype of the brooding, aloof romantic hero into which we often force him. Far from being above the "real" world, Galois was deeply concerned with justice and politics for much of his life. Galois's distrust of autocratic rulers and ossified institutions parallels his disdain for how establishment mathematicians practiced and taught mathematics.

Évariste Galois was born on October 25, 1811 in the village of Bourg-la-Reine, a few kilometers to the south of Paris. He lived in the house in which his grandfather had founded a boys' school. Jacques Olivier Galois's school had flourished during the French Revolution in 1789 since it was one of the few schools that did not belong to the priests. Both of Jacques's sons supported Napoléon Bonaparte, who rose to power after the Revolution and became Emperor in 1804. Whereas Théodore Michel fought in Napoléon's Imperial Guard, Évariste's father, Nicolas-Gabriel, inherited the school from Jacques [9, 200].

Nicolas-Gabriel became a leader of the liberal political party in Bourg-la-Reine during the First Restoration, when Napoléon was exiled to Elba in April of 1814, a few years after Évariste's birth. When Napoléon returned to Paris and took back control of France in March of 1815, a period later known as his Hundred Days, he appointed Nicolas-Gabriel mayor of Bourg-la-Reine. Upon Napoléon's return, however, many European states including Prussia and Britain allied against him, ultimately defeating him in the Battle of Waterloo in June. Nicolas-Gabriel was then supposed to return his post to his predecessor, but the latter was forced to leave the country over a mysterious scandal [9, 201]. Nicolas-Gabriel held his position for lack of other candidates, remaining mayor of Bourg-la-Reine for the rest of his life. On February 24, 1808, Nicolas-Gabriel married Évariste's mother, Adélaïde-Marie Demante, whose family lived across the street from his. Intelligent and generous, she taught her passion for honor and justice to her son. Indeed, she was Évariste's sole teacher until he was twelve years of old [9, 202].

Young Galois's mother finally let him attend school in 1823 at *Louis-le-Grand*, one of the most prestigious secondary schools in France. Galois did well during his first couple of years, earning first prize in Latin verse and three certificates. However, by the third year, he began to show a distaste for schoolwork. The principal wrote to Galois's father that the boy should repeat his third year. His father opposed this at first but ultimately acquiesced. While redoing his third year, Galois was allowed to take the next year of mathematics courses, and it was then that he discovered his extraordinary mathematical talent. Dissatisfied with the elementary algebra textbooks, Galois sought out the works of the inventors themselves: in short order he read through the works of Legendre and Lagrange. In the words of Dupuy, Galois's intelligence "left the plains to immediately scale the mountaintops" [9, 206].

Galois's family and teachers began to notice a change in his behavior. While his schoolmaster had originally found Galois "sweet, full of innocence and good qualities" at the beginning of the year, Galois became rebellious and bizarre. His rhetoric professors lamented that "his intellect appears to be only a legend that one will soon cease to believe" and that "in his work, when he deigns to do it, there is nothing but oddness and negligence" [9, 207]. His schoolmaster saw Galois's talent and implored him to divide his time between both math and letters, but to no avail. Defeated, he remarked that "the furor of mathematics possesses him" [9, 208].

After Galois failed his first entry exam to the esteemed *École Polytechnique*, he began a specialty math course under professor Louis-Paul-Émile Richard. Though Richard himself never published mathematical work, he was the mentor of other great mathematicians such as Joseph Serret and Charles Hermite [37, 450]. Richard was the only professor who recognized Galois's brilliance, going so far as to declare that the *École Polytechnique* should accept Galois without the entry exam, and under him Galois flourished. Galois published his first paper, on periodic fractions, in the *Annales de Gorgonne* in March 1st, 1829 at the age of seventeen [9, 209]. He even submitted his work to the *Académie des Sciences*. One of the foremost professors at the *École Polytechnique*, Augustin-Louis Cauchy, was supposed to present this work, but he delayed his presentation and ultimately cancelled it [32, 87]. (I examine this incident more closely in Chapter 6.)

Galois suffered two more personal disasters in July of that year. A fierce legislative election in 1827 had heightened political tensions, and Galois's father, who sought to maintain Bourg-la-Reine's relative independence from central authority, had a political enemy in the curate of Bourg-la-Reine. The curate circulated licentious epigrams forged and signed with the mayor's name. The resulting scandal forced Galois's father out of Bourg-la-Reine to an apartment in Paris, and on July 2 he asphyxiated himself. *Louis-le-Grand* was right next to his apartment. Young Galois led the mourners from Étienne-du-Mont in Paris to a cemetery in Bourg-la-Reine. In Bourg-la-Reine the villagers took their mayor's body from the hearse and carried it for over a mile. When they passed in front of the church where the clergy, including the curate, awaited the procession, they rioted, attacking the curate with both insults and stones. Amid this political turmoil young Galois watched his father's burial [9, 212].

A few days later, Galois attempted the entrance examination to the *École Polytechnique* again. He failed. Bell and other writers on the event have vilified the proctor, but Galois himself was not in a good mental state to take the exam, and, as we shall see, clarity in presentation was not his strong suit. Regardless of who was to blame, this failure crushed Galois's hopes of attending the *École Polytechnique*. In January of 1830, Galois enrolled in the less-prestigious *École Préparatoire* [9, 212]. Galois suffered another mathematical failure later that year. In February, he submitted work to the *Académie des Sciences* for the *Grand Prix de Mathématiques*, which would be decided in June that year [36, 137]. Unfortunately, Joseph Fourier, the referee assigned to judge Galois's work, died before examining it, and the papers were lost [9, 217].

3.3 Galois's politics

The year 1830 saw several events in Galois's life, such as his expulsion from the *École Préparatoire* and his joining the *Société des Amis du Peuple*, which led him down the path to political radicalization, to two arrests, and ultimately to his death in 1832. While we remember him for his mathematics, politics was Galois's other passion, and he turned to it increasingly as he suffered several mathematical failures. A discussion of his life would be incomplete without it.

Throughout his life Galois had absorbed the ideals of the French Revolution. He grew up in a house which had been one of the few boarding schools not under control of the Church before the French Revolution. His father and his uncle both supported Napoléon, with the former becoming the leader of liberalism in Bourg-la-Reine and the latter fighting in the army under Napoléon. Even Louis-le-Grand, despite its prison-like appearance, housed lively and rebellious students. It was no stranger to student revolts and between 1815 and 1823 it had gone through two principals [9, 203]. The students at *Louis-le-Grand* admired the *École Polytechnique* because it embodied the principles of the French Revolution. Most of the *École Polytechnique*'s students held strong liberal views, the entrance examination was open to all, its motto was "For nation, science, and glory." The *École Polytechnique*'s reputation for rigorous education was thus not the only factor that drew Galois; it stood for the very values that he himself held dear. For Galois, the *École Polytechnique* was perfect. Galois chafed under the authoritarian rule at *Louis-le-Grand*, and both the loss of his father to a malicious campaign by the Church and Galois's second failure of the exam for the *École Polytechnique* increased his hatred for authority. It was while he attended the *École Préparatoire* in 1830 that Galois began to act on his strong political convictions, resulting in his expulsion from the school by the end of the year.

On July 25, 1830, Charles X passed the July Ordinances, which, among other things, suspended the freedom of the press and dissolved the newly-elected Parliiament. Riots followed on July 27, lasting "Three Glorious Days." Among the rioters were students from the *École Polytechnique*, dressed proudly in their school uniforms and armed with rapiers. Galois and his fellow students at the *École Préparatoire* could only enviously watch them from behind barred windows. The school's director, Joseph-Daniel Guigniault, forbade the students to leave the building. Galois attempted to climb the walls of the *École Préparatoire* to join the revolution but to no avail. When it ended, he was still trapped in school. Even more outrageous for Galois, Guigniault then offered the services of his students to the new provisional government, [9, 220]. Galois criticized Guigniault's hypocrisy during the July Revolution in a letter to a newspaper later that year.

During the summer holidays Galois befriended young republicans such as Auguste Blanqui and Franois-Vincent Raspail, who would go on to become famous revolutionary Frenchmen. They, along with Galois, joined the *Société des Amis du Peuple*, a patriotic organization formed on July 30, and which had tried to prevent the rise of the new King Louis-Philippe immediately following the Three Glorious Days. The organization started out preaching their ideas in public. On September 25, however, during a public session a crowd with many members of the national guard attacked them while crying out "down with the clubs!" From that point on, the *Société des Amis du Peuple* held its meetings in private, though anyone was allowed to attend [38, 80].

The conservative regime of Louis-Philippe enabled Guigniault to run the*École Normale* in accord with his political views (he had reverted the *École Préparatoire* to its original name on August 8). Galois, by contrast, asked Guigniault if the students at the *École Normale* could wear uniforms like those at the *École Polytechnique* and if they could be armed so that they could do military training, questions that irked the politically conservative Guigniault. Galois also criticized Guigniault's decision to extend coursework at the school to last not two years but three. Galois's rebellious behavior isolated him from most of his peers, who did not want to draw the ire of the director. The conflict between Galois and the director came to a head in early December when a newspaper for students, *Le Lycée*, published a letter Guigniault had written criticizing a teacher at *Loius-le-Grand* [9, 225–226]. *Le Lycée*'s rival newspaper, *Gazette des Écoles*, published a letter by Galois which began: "The letter from M. Guigniault in yesterday's *Lycée*, on the occasion of an article in your newspaper, seemed to be quite out of place. I thought you would be interested in any attempt to unmask this man" [5, 462].

Galois then exposed the director as a conservative traitor to the cause of the July Revolution. He wrote that Guigniault had threatened to call the police to keep the students contained in the school on July 28, a request that would have been absurd during the riots. Galois also noted that during the fighting Guigniault had remarked that, if he were a soldier, he would not know whether to sacrifice freedom or legitimacy–whereas a good republican would surely have liberty. Galois strongly suggested that Guigniault was a hypocrite for his about-face after the protesters' victory and accused the director of simply trying to make the school resemble the former *École Normale*, concluding that "everything he does shows his narrow outlook and ingrained conservatism." *La Gazette* removed Galois's signature (though not at Galois's request) and added its own snippet emphasizing Guigniault's hypocrisy. Galois neither confirmed nor denied his authorship [9, 230–231], but there was little doubt as to who had written the letter. Galois himself had told several of his classmates that he intended to write such a letter [9, 227]. Galois was expelled.

Soon after his expulsion, the *Académie* mathematician Siméon-Denis Poisson contacted Galois and encouraged him to re-submit the manuscript Galois had submitted to the *Grand Prix de Mathématiques*, and Galois agreed. Galois also turned to private teaching in a bookshop after his expulsion on January 4 of 1831. On January 13, 1831 he advertised it as a course presenting "new theoretical aspects, none of which have yet been published or been the subject of public lectures," apparently referring to his own mathematical work [9, 233]. About forty people showed up on the first day, but the course did not last long. Then Galois received notification that Poisson and the other referee, Sylvestre Lacroix, could not comprehend Galois's paper and thus rejected it. (They did in fact encourage him to clarify his ideas and resubmit the memoir. I will discuss this further in Chapter 6.) Up to this point Galois had already been working with the *Société des Amis du Peuple*, and this rejection drove him further into politics [9, 234].

In early May that year, Galois was arrested for threatening the King's life at a republican banquet, and he was put on trial on June 15. The June 16, 1831 edition of the *Gazette des Tribunaux* [14] published the transcript for Galois's trial. It provides a summary of what transpired at the banquet and contains Galois's testimony. I describe it in detail here because it gives one a good sense of Galois's personality and of the political climate in which he lived.

The banquet took place on May 9, 1831, to celebrate the acquittal of nineteen artillerymen who were accused of conspiring against the state. Around 200 republicans attended this banquet. At one point, people began to offer toasts such as "to the 1793 revolution!" and "to Robespierre!" and "to the Mountain!" (The Mountain was a radical political party that, led by Maximilien Robespierre, wrought the Reign of Terror in 1793.) These toasts were followed with cries such as "long live the Republic!" and "long live the Mountain!" Some of the guests were not satisfied with the Revolution of 1830. One cried "to the sun of July 1831! Let it be as warm as that of 1830!", in reference to the Trois Glorieuses. The republicans were itching for a revolution: according to Galois, this cry was followed with "Sooner! Sooner!"

Galois sat to the left of the host at the end of the banquet hall. He took the knife or dagger–the transcript ambiguously calls it a "knife-dagger"–with which he was eating, and holding also his glass in the same hand, he raised them up and gave the toast "To Louis-Philippe!" Many people hissed–possibly, in the court's view, because people were disavowing this threat on the King's life. In Galois's view, they hissed because they initially thought he was toasting to the King's health. As the republicans learned the true meaning of Galois's toast, several imitated him and others cheered.

In his testimony, Galois confirmed these details. He said, however, that his toast was "To Louis Philippe, *if he betrays*." The noise of the audience had muffled this qualifying phrase. In his testimony Galois did little to help prove his innocence. When the presiding judge asked Galois if he had intended his toast as a provocation, Galois replied: "Certainly, it was a provocation in the case where Louis-Philippe betrays." The judge then asked if Galois believed the King would betray. To Galois, it was certain: "Everyone anticipates it; the way things are going in the government could lead one to believe that Louis-Philippe is capable of betraying the nation because he has not given us enough of a guarantee of his sincerity to make us not fear this result."

The judge asked Galois to elaborate, but when Galois began to do so his attorney fortunately cut him off. Next, the judge asked Galois why he had brought his knife (or dagger) to the banquet. Galois had brought it "by chance. I've carried it with me every day since I bought it." As to why he had had it made in the shape of a dagger, Galois quipped that "it is with similar knives that republicans cut their turkey and chicken."

After this, the witnesses were called forward. The first six witnesses, as recorded in the transcript, offered little of interest. The seventh, the novelist Gustave Drouineau, refused to take the oath in front of the court. He then declined to speak on what had taken place at the banquet, saying that it had been a private event so he was not obligated to say what he had seen. When the prosecutor objected that he was required by law to disclose the information, Drouineau responded that "there is a law that is to me more sacred than those written on perishable paper: it is the law of honor." The court gave him a fine of 100 francs but allowed him to leave.

After Drouineau, the witnesses in defense of Galois came to the stand, confirming that they had heard him say "if he betrays" after his toast. The prosecutor maintained that the banquet was a public gathering and pointed out that when Galois was first taken in, Galois had not mentioned that he had used the phrase "if he betrays." To this, Galois confessed that

> there may have been a little mischief on my part: you would not imagine the joy of the police commissioner when he thought he had caught a conspirator. He almost did not believe his luck; he must be a little dismayed. [13, 774]

He then objected to the claim that the King would not betray the nation:

> No one today is gullible enough to believe that a king is perfect, especially after the judges who, under Charles X, pursued us for having said that a king could fail, took an oath to another who was placed on the throne because of the stupidity of that decaying king. [13, 774]

Galois read a speech (which the *Gazette des Tribunaux* did not transcribe) "full of exaltation" in which he declared that he was among those who for eight months had marched several times through the streets with weapons. He added that he had wished to be with his friends in the audience of a conspiracy case the previous Saturday in which they had insulted the witnesses, the judges, and the jury [9, 237]. The judge gently interrupted him (in a manner which Dupuy describes as "paternal") observing that Galois was harming his own defense. He then gave the floor to Galois's lawyer. His lawyer merely maintained that all of this happened in a private setting. The judge and jury alike appeared to have been moved by Galois's youth; the *Gazette des Tribunaux* describes the judge ending his summary of the case with "an appeal to the sentiments of the jury as fathers of families." After 10 minutes of deliberation, the jury found Galois not guilty. Galois took back his knife and left the court [9, 238].

Galois did not stay out of trouble for long, however, as he was preparing for a republican patriotic demonstration on July 14, 1831 [9, 238]. On that same day, back in 1789, the citizens of Paris stormed the Bastille, a state prison and a symbol of the oppressive monarchy. In this they hoped to gain ammunition and gunpowder and to free political prisoners locked within (of which there were only seven at the time). This ignited the French Revolution in 1789, hence it had great significance for the republicans. Galois and his friend Vincent Duchâtelet led the demonstrators. Galois, for his part, wore his National Guard uniform and was heavily armed. Galois and Duchtelet were arrested. While they waited in jail, Duchtelet drew on his cell wall the King's head next to a guillotine with the words "Philippe will take his head to your altar, O Freedom!" [9, 238].

Raspail happened to be arrested at the same time as Galois, and what we know of Galois's time in prison we know through Raspail's writings [32, 94]. The two were held at the Sainte-Pélagie, which held many other revolutionary prisoners, both then and during the French Revolution. The republican prisoners, including Galois, would hold evening ceremonies during which they sang patriotic songs. In late July, some of the prisoners taunted Galois into drinking liquor, with which he had little experience. Raspail wrote that Galois got drunk and confided in him:

> I do not like women and it seems to me that I could only love a Tarpeia or a Graccha [two women from Roman legend; the former betrayed Rome to the Sabines in exchange for jewelry, the latter refers to Cornelia Gracchus, the mother of Tiberius]. And I tell you, I will die in a duel on the occasion of some *coquette de bas étage*. Why? Because she will invite me to avenge her honor which another has compromised. Do you know what I lack, my friend? I confide it only to you: it is someone whom I can love and love only in spirit. I have lost my father and no one has ever replaced him, do you hear me? [30, 89]

After this, Galois reportedly cried out "You despise me, you who are my friend! You are right, but I who committed such a crime must kill myself!" He then tried to do just that, but Raspail and other prisoners restrained him [30, 90]. The comment about the *coquette de bas étage* apparently foreshadows Galois's death in a duel the following year, but we may doubt its credibility, as Raspail published his book in 1839, seven years after Galois's death, and with knowledge of its circumstances. A few days later, on July 29, one year after the July Revolution, a shot was fired toward the prison building from the attic of a house across the street where one of the prison guards lived. Galois was in the room at this time, and reported where the shot had come from. According to Dupuy, "perhaps he added some cold insult to the director, as he had addressed the judges in his first trial," as he was immediately thrown in the dungeon [9, 243]. The prisoners were indignant at this punishment of their "little scholar" [30, 118]. They rose up and took control of the prison until the army was called in [32, 96].

3.4 Stéphanie Dumotel

Months later, during the spring of 1832, a cholera epidemic swept through Paris, and Galois was transferred to the Sieur Faultrier clinic on March 16, 1832. There, he fell in love with a young lady, over whom he would eventually die in a duel. Galois wrote her name, "Stéphanie," in the margins of some of his manuscripts. In the margins of his *Discours Préliminaire*, for example, he wrote her name several times intertwined with his. In other parts of the document, he wrote their initials together, "E.S.", and, at the bottom of the first page, he wrote what Bourgne and Azra call "dreams or happy hopes": "SUPERBE STÉPHANIE, GALOIS SUPERBE." They also note that "this charming name became that of the 'infamous coquette' who was the cause of his duel '' [5, 501–502].

Galois also copied two letters from Stéphanie. Bourgne and Azra observe that Galois "crossed them out with long streaks and, it must be admitted, with a rare violence" [5, 489]. The first letter is dated May 14, "183"; the missing last digit should be a "2." I have left the crossings-out as blank spaces, and I have left some isolated words in French. The first letter reads

> Let us break up this affair I pray you.   I do not have enough spirit to follow   a correspondence of this kind   but I will try to have enough to converse with you as I did before anything happened. Here is   Mr. *le*

> *en a* *qui* *doit vous* *qu'a* me and don't think anymore about things which could not have existed and which never will exist.
>
> Mademoiselle Stéphanie D.
>
> 14 May 183– [5, 489]

Galois crossed out the last letters of the signature to hide Stéphanie's family name. The ink of the crossings-out, spread with the base of a pen-holder, was from Galois's last night [5, 490–491]. Some verbs are without a subject but from their conjugation we can try to infer their subjects and direct objects–I have indicated my guesses in square brackets. Some words have also been underlined, though by whom we do not know. The second letter, undated, reads

> I have followed your advice and I have thought over what has happened under whichever denomination it may have happened between us. In any case, Sir, be assured that there, without a doubt, never would have been more. You assume wrongly and your regrets are ill-founded. True friendship exists scaracely exists except between people of the same sex particularly of friends. sorry [for you] in the emptiness that the absence of all feeling of this kind ... my trust ... but it has been very wounded ... you have seen me sad you asked [me] the reason; I answered you that I had sorrows that one had made me suffer. I thought that you would handle that as anyone in front of whom one drops a word for these one is not The calm of my opinions lets me judge without much reflection the people I usually see; this is why I rarely have the regret of being mistaken or influenced concerning them. I am not of your opinion *les sen* more than *les a* *exiger* *ni se* *vous remercie sincèrement de tous ceux ou vous voudriez bien descendre en ma faveur.* [5, 490]

I am not certain what to make of the last sentence. Were it not for the word "*ou*", which could only mean "or" or be part of an "either...or" construction, it would seem to read: "[I] thank you sincerely for all those *ou* [whom] you would like to bring down in my favor," a statement which would be of great relevance to understanding why Galois died in a duel. If the word is indeed "ou", however, this sentence makes little sense: "...thank you sincerely for all those *or* you would like to go down in my favor." In this later case with "or," the word "descendre" loses its direct object which could be filled in by replacing "ou" with "que" ("...those *whom* you would like to bring down in my favor.") Nevertheless, Bourgne and Azra wrote "ou". If this is the case, then it seems that Galois crossed out an essential part of the sentence. If, instead, the "ou" is a typo on the part of Bourgne and Azra and should have instead been "que," Stéphanie might take on a more active role in the duel: rather than be the simply the object of the duel (as I will argue in the next section), she would have *encouraged* Galois in initiating the duel.

In 1968, Carlos Infantozzi investigated Stéphanie's identity further. Upon examining the original first letter with a magnifying glass, he determined Stéphanie's last name: Dumotel. From his archival research, we know that her full name was Stéphanie-Félicie Poterin du Motel. She was the daughter of a physician at the Sieur Faultrier, Jean-Louis Auguste Poterin du Motel. In 1840, eight years after Galois's death, she married a language professor [18, 157]

Galois was released from prison on April 29, 1832 that year. He received the first letter from Stéphanie on May 14. A couple weeks later, on May 30, Galois was shot in a pistol duel. He died the next day.

4 The duel

On June 4, 1832, the French newspaper *Le Précurseur*, briefly mentioned Galois's death:

> Un duel deplorable a enlevé hier aux sciences exactes un jeune homme qui donnait les plus hautes espérances, et dont la célébrite précoce, ne rappelle cependant que des souvenirs politiques. Le jeune Évariste Gallois, condamné il y a un an pour des propos tenus au banquet des Vendanges de Bourgogne, s'est battu avec un de ses anciens amis, tout jeune homme comme lui, comme lui member de la société des Amis du Peuple, et qui avait, pour dernier rapport avec lui, d'avoir figuré également dans un procès politique. On dit que l'amour a été la cause du combat. Le pistolet étant l'arme choisie par les deux adversaires, ils ont trouvé trop dur pour leur ancienne amitié d'avoir à viser l'un sur l'autre, et ils s'en sont remis à l'aveugle decision du sort. A bout pourtant, chacun d'eux a été armé d'un pistolet, et a fait feu. Une seule de ces armes était chargée. Gallois a été percé d'outre en outre par la balle de son adversaire; on l'a transporté à l'hpital Cochin, où il est mort au bout de deux heures. Il était âgé de 22 ans. L.D., son adversaire, est un peu plus jeune encore. [28, 3]

> [A deplorable duel yesterday deprived the exact sciences of a young man who had the greatest potential, and who had a precocious fame, but only for political incidents. The young Évariste Gallois [sic], convicted a year ago for matters that took place at a banquet at Vendanges de Bourgogne, fought

> with one of his old friends, a young man like himself, and like him a member of the Société des Amis du Peuple, and who had, in another similarity with him, been also involved in a political trial. It is said that love was the cause of the battle. The pistol was the chosen weapon of the two adversaries, they found it too difficult to aim at each other because of their old friendship, and left it up to the blind choice of fate. Yet each was armed with a pistol and fired. Only one of the pistols was loaded. Gallois [sic] was pierced through by the bullet of his adversary; he was transported to the Cochin hospital, where he died after two hours. He was 22 years old. L.D., his adversary, is a bit younger.]

This article is not entirely accurate. For example, Galois was 20 years old, not 22 years old, when he died. Moreover, we do not have other sources to confirm the exact circumstances surrounding the duel. This has led many writers and historians to propose their own theories about the duel, its cause, and Galois's killer. Several such authors have attached various political meanings to the duel. The writer Tony Rothman has convincingly disproved the arguments of some of these authors: Leopold Infeld, Fred Hoyle, and E.T. Bell. One historian, Laura Toti Rigatelli, argued after Rothman, in 1996, that Galois used the duel as a form of political suicide in an attempt to spark a revolution. This is the main account that I intend to refute in this chapter. The evidence that exists, which consists of Galois's letters, newspaper articles, and memoir snippets, fails to support or even directly contradicts these theories. This evidence *does* indicate the identity of Galois's killer, Pescheux d'Herbinville, as well as a plausible cause. As the article in *Le Précurseur* reports, love seems to have caused this duel.

On May 25, 1832, a few days before the duel, Galois lamented the loss of Stéphanie du Motel in a letter to his close friend, Auguste Chevalier:

> Comment se consoler d'avoir épuisé en un mois la plus belle source de bonheur qui soit dans l'homme, de l'avor épuisé sans bonheur, sans espoir, sr qu'on est de l'avoir mise à sec pour la vie? [5, 468]
>
> [How can one recover after having exhausted in a month the greatest source of happiness man has, having exhausted it without happiness or hope, sure that one is to be left dry for life?]

In this paragraph, the hopelessness Galois felt is clear and might lend credence to the hypothesis that he sought the duel as a form of suicide. In the next paragraph, he indeed expressed a strong urge to act violently:

> Oh ! venez après cela prcher la paix ! venez demander aux hommes qui sentent d'avoir pitié de ce qui est ! Pitié, jamais ! haine, voilà tout. Qui ne la resent pas profondément, cette haine du présent, n'a pas vraiment l'amour de l'avenir. [5, 468]
>
> [Oh! go after that to preach about peace! go ask what it is to men who feel that they have pity! Pity, never! hate, that's all. He who doesn't feel it deeply, that hate of the present, doesn't truly have love for the future.]

Here Galois suggested he still has hope for the future, hope that it would be better than the hateful present in which he lived. Galois told his friend Chevalier about his desire for revenge:

> Quand la violence ne serait pas une nécessité dans ma conviction, elle le serait dans mon cuer. Je ne veux pas avoir souffert sans me venger.
>
> A part cela, je serais des vôtres. [5, 468]
>
> [When violence will not be a necessity in my conviction, it will yet be a necessity in my heart. I don't want to suffer without avenging myself.
>
> In that respect, I agree with you.]

However, he didn't see vengeance as his ultimate goal:

> Mais laissons cela; il y a des êtres destinés peut-être à faire le bien, mais à l'éprouver, jamais. Je crois être du nombre. [5, 468]
>
> [But let's leave that, there are beings who are maybe destined to do good, but to never experience it. I believe that I'm one of them.]

This contradicts the claim that Galois had hoped to commit suicide in a duel. True, he had lost the love of his life, and academia seemed to have rejected him. Although he had almost nothing left to lose, he might still have found something to gain in this world. The next two paragraphs affirm this:

> Tu me dis que ceux qui m'aiment doivent m'aider à aplanir les difficultés que m'offre le monde. Ceux qui m'aiment sont, comme to [tou? tu?] le sais, bien rares. Cela veut dire, de ta part, que tu te crois, quant à toi, oblige à faire de ton mieux pour me convertir. Mais il est de mon devoir de te prévenir, comme je l'ai fait cent fois, de la vanité de tes efforts.

> J'aime à douter de ta cruelle prophétie quand tu me dis que je ne travaillerai plus. Mais j'avoue qu'elle n'est pas sans vraisemblance. Il me manqué, pour tre un savant, de n'tre que cela. Le cur chez moi s'est révolté contre la tte; je n'ajoute pas comme toi : C'est bien dommage [5, 468]
>
> [You tell me that those who love me should help me smooth out the difficulties that the world gives me. Those who love me, as you know, are very few. That means that you feel, in regards to yourself, obligated to do your best to convert me. But it is my duty to warn you, as I've done a hundred times, of the futility of your efforts.
>
> I would love to doubt your cruel prophecy of when you told me that I will not work again. But I admit that it does not lack verisimilitude. I only want to be a savant. My heart revolts against my head; I do not add like you: it's a pity.]

Galois realized that, in spite of his own reason, he believed that he might still have a future in academia. In the following sentences, not only does Galois give no indication that he hoped to die in a duel, but he also promised to see Chevalier soon and described a clear next step for his life:

> J'irai te voir le 1er juin. J'espère que nous nous verrons souvent pendant la première quinzaine de juin. Je partirai vers le 15 pour le Dauphiné [5, 469]
>
> [I'll come to see you on June 1. I hope that we'll see each other often during the first half of June. Around the 15th, I will leave for the Dauphiné...]

It seems implausible that someone seriously contemplating suicide would plan to move somewhere new in a few weeks. Though we don't know for sure Galois's reasons for wanting to go to Dauphiné, a province in southeastern France at the time, it was a region of strong industrial development during the 19th century [4, 354]. This made it a reasonable option for someone looking for work. Even better for the young republican, its people had an "egalitarian spirit" and had never appreciated the Restoration [4, 339; 368]. Galois left Chevalier the following postscript:

> En relisant ta lettre, je remarque une phrase où tu m'accuses d'être enivré par la fange putréfiée d'un monde pourri qui me souille le cœur, la tête et les mains.
>
> Il n'y a pas de reproches plus énergiques dans le repertoire des hommes de violence.

De l'ivresse ! Je suis désenchanté de tout, mme de l'amour de la gloire. Comment un monde que je déteste pourrait-il me souiller ? Réfléchis bien. [5, 469]

[Upon rereading your letter, I noticed the phrase where you accuse me of being intoxicated by the putrid atmosphere of a rotten world that has soiled my heart, mind, and hands.

There are no reproaches more vigorous in the repertoire of men of violence.

Intoxicated! I am disenchanted with everything, even with the love of glory. How can a world that I hate soil me? Reflect on it well.]

This postscript reiterates the theme of this letter: though Galois was deeply dissatisfied with the world, he didn't see himself reacting violently as, say, Chevalier might. His losses had made him resentful and disheartened, but he thus had little left to lose. He believed that, rather than violently throw away his own life, he might yet do some good in the world. These are not the words of a man who hopes or expects to die in a duel less than a week later.

Nevertheless, a few days later, on May 29, Galois wrote a letter addressed to "all republicans" (according to Chevalier, [7, 753]), declaring the he would die in a duel the following day:

Je prie les patriotes, mes amis, de ne pas me reprocher de mourir autrement que pour le pays.

Je meurs victime d'une coquette, et de deux dupes de cette coquette. C'est dans un miserable cancan que s'éteint ma vie.

Oh ! pourquoi mourir pour si peu de chose, mourir pour quelque chose d'aussi méprisable!

Je prends le ciel à témoin que c'est constraint et force que j'ai cédé à une provocation que j'a conjurée par tous les moyens.

Je me repens d'avoir dit une vérité funeste à des hommes si peu en état de l'entendre de sang-froid. Mais enfin j'ai dit la vérité. J'emporte au tombeau une conscience net de mensonge, nette de sang patriote.

Adieu ! j'avais bien de la vie pour le bien public.

Pardon pour ceux qui m'ont tué, ils sont de bonne foi.

E. Galois [5, 471]

[I pray that you patriots, my friends, won't reproach me for dying for something other than for the country.

I die the victim of a coquette [flirtatious woman], and of two dupes of that coquette. My life ends in a miserable *cancan*.

> Oh! Why die for something so small, to die for something so contemptible?
>
> Heaven is my witness that I am forced to surrender to a provocation that by every means I've tried to avert.
>
> I regret having said such a fateful truth to men who were incapable of hearing it rationally. Nevertheless, I have said the truth. I will take with me to the grave a conscience free of lies and free of patriot blood.
>
> Farewell! I lived a good life for the public good.
>
> Forgive those who kill me, for they are of good faith.]

In their introduction, Bourgne and Azra wrote that Galois's papers, including his manuscripts and the Testamentary Letter to Chevalier, were found in a pile on a table in his room at the Sieur Faultrier. In his eulogy of Galois that he wrote a few months later, Chevalier said that it was Galois's mother who had brought him the manuscripts and the Testamentary Letter in the end of June that year [7, 753]. Chevalier also printed the letters that Galois wrote to "all republicans" in this eulogy. While he does not specifically say whether Galois sent these letters as well, if he had done so it is unclear why he would not have sent the Testamentary Letter to Chevalier as well.

Galois wrote that he would die over a woman. The word *cancan*, which I have left untranslated, roughly means "gossip" and here implies the triviality of the duel's cause. We should interpret this letter with dueling custom in mind. Although dueling traditions changed over the centuries and varied in European countries, there still were certain common principles. It was not uncommon for a gentleman to die in some "miserable *cancan*." It was particularly egregious to insult a lady, and in such a case a male friend or suitor would likely challenge the offender to a duel [1, 32]. An Irish treatise written in 1777 on dueling (and quoted in Baldick) states:

> X. Any insult to a lady under a gentleman's care of protection to be considered as by one degree a greater offence than if given to the gentleman personally, and to be regarded accordingly. [1, 35]

Though written specifically for Irish dueling, the rules it established matched those on the Continent [1, 33–34]. Galois implied in his letter that he had offended a woman's honor, provoking two men, or "dupes" as he calls them, attached to her. This lady was Stéphanie du Motel. The Irish treatise deemed "giving the lie" and hitting a man "the two greatest offences" [1, 35]. Given the severity of the former offense, men often fought duels over each other's word. This helps explain Galois's insistence that he has spoken the truth, whatever that truth may be.

Despite the gravity of injuring the honor of a lady or a gentleman, it was possible to avert a duel after the challenge had been given. Rule XX of the Irish treatise states:

> XX. [Men known as] Seconds are bound to attempt a reconciliation [of the duelers] before the meeting takes place, or after sufficient firing or hits as specified. [1, 36]

Thus, when Galois said he had attempted to avert the duel, those attempts fell within standard dueling conduct. His duel curiously lacked the presence of a "second" on either side (though its possible that the "two dupes" referred to the offended and his second). In fact, Galois's duel did not exactly follow the standard dueling conduct. Contemporary sources disagreed over the minutiae of a proper duel. Chateauvillard's *Essai sur le Duel*, a treatise on dueling written in 1836 and accepted by many authorities (Chateauvillard included in his essay the list of officials who had approved of it) talks of several "witnesses" in addition to the primary contenders and their seconds. The Irish treatise, on the other hand, made little mention of these witnesses. In Chateauvillard's view, the witnesses took on some of the duties of the seconds in the Irish treatise, such as attempting reconciliation before the duel and arranging for a surgeon to be present.

At the same time, Galois's duel was not entirely anomalous. Chateauvillard wrote a small chapter on duels in which only one of the pistols is loaded, making clear from the outset that it was only acceptable under "extraordinary" circumstances and that neither party ever had to agree to it [6, 78]. In this event, the random drawing of lots determined who would use the loaded gun, though the witnesses would prime both weapons [6, 79]. In this case again, Chateauvillard required the witnesses to take a surgeon along with them to the duel (which Galois and his opponent failed to do).

We should not turn to these treatises on dueling to deduce the exact proceedings of any contemporary duel. They capture instead the general standard of the times, from which any individual duel might have varied. Accusing someone of being a liar or insulting a lady were offenses often deemed worthy of death. Duels such as Galois's traditionally took place at dawn which helped to ensure the least publicity" [21, 146]. A man's honor had to be upheld at all costs, regardless of the frivolity of the offense. To reject a duel was worse than death. Victor Kiernan illustrated this in his book, *The Duel in European History*, with a vignette:

> A gentleman of Bordeaux, on the eve of the French Revolution, who refused a challenge on account of religious scruples, was so cruelly taunted and baited that at last in desperation he turned on one of his tormentors, and was killed. [21, 157]

With these dueling customs in mind we turn to the cause of Galois's duel. The temperamental Galois could easily have offended du Motel after she had broken off their courtship and provoked "two dupes" who were also attached to her. It is not clear what "fateful truth" Galois said to the men protecting du Motel's honor, but his insistence on speaking the truth and not lies exemplifies a grave attitude towards lying and being deemed a liar. Perhaps Galois's fierce persistence foiled any attempts at reconciliation.

Galois wrote another letter the night before the duel, this time to two republicans to whom he was particularly close [7, 753], Napoléon Lebon and V. Delaunay:

> Mes bons amis,
>
> J'ai été provoqué par deux patriotes. Il m'a été impossible de refuser.
>
> Je vous demande pardon de n'avoir averti ni l'un ni l'autre de vous.
>
> Mais mes adversaires m'avaient sommé SUR L'HONNEUR de ne prévenir aucun patriote.
>
> Votre tâche est bien simple: prouver que je me suis battu malgré moi, c'est-à-dire après avoir épuisé tout moyen d'accommodement, et dire si je suis capable de mentir, de mentir mme pour un si petit objet que celui dont il s'agissait.
>
> Gardez mon souvenir, puisque le sort ne m'a pas donné assez de vie pour que la patrie sache mon nom.
>
> Je meurs votre ami. [5, 471]
>
> [My dear friends,
>
> I have been provoked by two patriots. It was impossible for me to refuse.
>
> Forgive me for not warning either one of you.
>
> But my adversaries have beseeched me ON MY HONOR to not warn any patriot.

> Your task is quite simple: prove that I fought despite myself, that is to say after having made every effort to compromise, and say whether I am capable of lying, of lying even about a subject as petty as this.
>
> Safeguard my memory, since fate has not given me enough life for the homeland to know my name.
>
> I die your friend.]

This letter shows the same preoccupation with truth and lying and essentially repeats the previous one. We see again Galois's insistence on his own honesty. We see also his own admission that he could not refuse the duel. His honor prevented him from even informing those close to him, lest they attempt to stop him. Again, this was a dueling norm. In neither of these letters did Galois identify his rivals. He only said that they are patriots "of good faith." The article in *Le Précurseur* gave the perpetrator the initials "L.D." In the memoir of the novellist Alexandre Dumas, a republican (though more moderate than those in the *Société des Amis du Peuple*), he identified Galois's killer as he writes about the scene at the banquet mentioned in the article:

> Un jeune homme, tenant de la même main son verre levé et un couteau-poignard ouvert, s'efforait de se faire entendre. C'était Évariste Gallois, lequel fut, depuis, tué en duel par Pescheux d'Herbinville, ce charmant jeune homme qui faisait des cartouches en papier de soie, nouées avec des faveurs roses.
>
> Évariste Gallois avait vingt-trois ou vingt-quatre ans à peine à cette époque; c'était un des plus ardents républicains. [8, 161]

> [A young man, holding in the same hand a raised glass and an unsheathed dagger, tried hard to make himself heard. He was Évariste Gallois [sic], who was, later, killed in a duel by Pescheux d'Herbinville, a charming young man who made bullet cartridges out of tissue paper tied with pink strings.
>
> Évariste Gallois was hardly twenty-three or twenty-four years old when he died at this time; he was one of the most ardent republicans.]

Pescheux d'Herbinville was one of the nineteen artilleryman who were on trial in April of 1831 described in the previous chapter. This fact aligns with *Le Précurseur*'s statement that Galois's killer was also involved in a political trial. It also explains why Galois would say that his opponents were "of good faith." There is a discrepancy between the memoir and the article, however: Pescheux d'Herbinville does not have the initials "L.D." The writer Olivier Courcelle points out that this inconsistency likely came from the fact that journalists sometimes added "Le" to the beginning of names. He cites, for example, a trial account in which d'Herbinville's name is written as M. Lepescheux" three times [13, 533]. In this same newspaper, d'Herbinville mentioned that he made his cartridges out of green satin paper; the pink satin cartridges that Dumas remembered were made by another gunner [13, 538]. (It should also be noted that in the latter pages of the *Gazette*, d'Herbinville's name is written M. Pescheux d'Herbinville.") Despite these inconsistencies, d'Herbinville is apparently the man who killed Galois. The identity of the second "dupe," presumably the "second" of d'Herbinville, remains a mystery.

The two letters Galois wrote to his fellow republicans seem to show some ambiguity as to who actually provoked the duel. In the first letter to "all republicans," Galois called Stéphanie a "*coquette*" and the whole affair a "miserable *cancan*." In this context, the "fateful truth" which Galois said to Stéphanie's "two dupes" seems to be what provoked the men to challenge Galois to the duel. In the second letter, however, Galois writes that *he* was "provoked" by the two men. Furthermore, in Stéphanie's second letter, she seems to have written "thank you for all those *that* you would bring down in my favor," assuming we may replace "*ou*" with "*que*." Finally, if we entertain the idea that Raspail, when quoting Galois in prison, had embellished the event to give a certain prescience to Galois's words, then his phrase "she will invite me to avenge her honor which another has compromised" certainly fits this scenario. All this could lead to the hypothesis that Galois died in a duel *defending* Stéphanie's honor, rather than the other way around.

These three points fail to explain, however, why Galois would call Stéphanie a "*coquette*" and his opponents "her two dupes," when presumably Galois is the "dupe" depending Stéphanie's honor. If Galois had felt that Stéphanie's honor was important enough to die in a duel over it, he would not have called it "small" or "contemptible," or the whole argument a "*cancan*" in the first letter. Galois wrote in this same letter that he was "forced to surrender to a provocation," but he had tried "by every means" to avert it as well." He would not call challenging someone to a duel "surrendering to a provocation," nor would he seek to avert a challenge that he proposed. Even in the second letter, Galois wrote that "it was impossible for me to refuse" the provocation by the two other patriots. He also wrote that they demanded “ON MY HONOR” to not tell anyone of the duel. This is hardly the kind of demand someone who had been challenged to a duel would make of the challenger. Finally, Galois wrote that “I fought despite myself, that is to say after having made every effort to compromise.” Again, if it had been up to Galois to make the challenge, he could have simply not done so. We should thus interpret Galois saying “I was provoked” as another way of saying “I was challenged to a duel” in this instance. This only leaves Stéphanie’s mysterious thanks to Galois at the end of her second letter. On this I can only say that we do not know what Galois might have said to Stéphanie to elicit such thanks. In Galois’s own letters, however, he clearly portrayed his two adversaries as the aggressors.

The duel was fought over Stéphanie du Motel, and Galois was the offender. A few writers, including E. T. Bell, Leopold Infeld, and Fred Hoylehave argued that Galois died for political reasons. Bell, for example, suggested that Galois “had run afoul of political enemies immediately after his release” [2, 375], and Infeld proposed that Stéphanie had in fact set up Galois for a duel with a police spy [19, 308–311]. Hoyle, on the other hand, argued that Galois’s compatriots thought that he was not fully aligned with the republican cause. Rothman critiqued each of these in his 1982 article. Though the police did use spies against the rebellious secret political societies of the time, Rothman claims that neither Dumas “nor...any republican in Paris ever held any suspicions that d’Herbinville was an agent” [32, 99]. That thousands of republicans attended Galois’s funeral affirms this [38, 91]. Rothman also uses the fact that d’Herbinville was himself an ardent republican to further reject Infeld’s theory. As for Hoyle’s claim that Galois’s republican friends distrusted him, Rothman replies:

> To suggest as Hoyle does that any republican in Paris suspected Galois after his expulsion from l’École Normale, his Artillery activities, his threat to the King, his arrest, trials, sentencings, resentencings, and prison activities borders on the fantastical. This is in addition to the fact two or three thousand republicans later attended the funeral of this supposed agent provocateur. One might equally well claim Lenin had been suspected of being a Menshevik. [32, 100]

Rothman convincingly rebuts the arguments of Bell, Infeld, and Hoyle, so I will not go into more detail concerning their arguments. Rothman's work was published in 1982, however, and since then another author, Laura Toti Rigatelli, has proposed another theory to explain the duel. We now turn to her account.

Rigatelli, in her biography of Galois, portrays the duel as a plot that Galois devised to spark a political uprising. The Société des Amis du Peuple sought to ignite a new revolution in May 1832. They felt confident in their success, according to Rigatelli, due to a potential weakness in Louis-Philippe's government: a potential heir with relations to the previous king, Charles X, had surfaced, and his mother, the Duchess of Berry and the widow of Charles X's assassinated son, had returned to France. The Société suspected that the legitimists, who stood against Louis-Philippe's rule and counted on this heir, would willingly fight the King. The Sociéte thus convened on May 7 to plan their revolution. At the same time, Galois had just been released from prison, and the Société invited him since "he was well-known for his ability to spur the more lukewarm spirits into action" [31, 109].

In order to incite others to join, the Société needed some sort of pretext. In Rigatelli's account, the republicans began to discuss avenging the corpse of a hero, "in whose name the people of Paris would fight, a name to shout, while firing on Louis-Philippe's police, a name on the lips of the dying" [31, 109]. Galois soon spoke up and volunteered himself as sacrifice. Though reluctant at first, his compatriots acquiesced, and they planned the uprising to happen at his funeral. At this funeral, however, as two leaders from the Société gave their speeches, the republicans learned that a more suitable sacrifice, Maximilien Lamarque, a general whom Napoléon had appointed Marshall of France, had passed the day before, and they rescheduled the uprising to the General's funeral. (His obituary appears on the same page as Galois's in *Le Précurseur*.) Thus, "Évariste Galois's death had been pointless" [31, 113].

I have found no evidence to support either Rigatelli's claims or her depiction of the events leading up to Galois's death, even after examining the sources in her bibliography. One such source, Georges Weill's *Histoire du Parti Républicain en France* (History of the Republican Party in France), written in 1900, does corroborate Rigatelli's claim that the republicans switched the date of the Revolution from the day of Galois's funeral to that of General Lamarque's. In Weill's version, however, the decision was made that morning rather than during the funeral. Weill also identified tension over the cholera epidemic, which many republicans believed was "propagated by poverty" [38, 90], as a cause for the riot on June 5, rather than the intrigue surrounding the Duchess of Berry's return. Missing from Rigatelli's sources is any evidence that Galois volunteered himself to be the republicans' sacrifice as well as the plan to stage his duel. Weill only mentioned that the republicans were waiting for a favorable moment to rebel and ended up choosing Galois's funeral. He did not specify when the members of the Société made this choice, but it is certainly plausible that they came to this decision shortly after Galois's death, given that they were simply waiting for the right moment for their revolt.

In her preface, Rigatelli writes that "On the basis of the analysis and interpretation of a series of hitherto neglected documents. [sic] I have been able to provide a new version of the circumstances leading to Galois' death" [31, 9]. She lists the following sources: the article mentioned above in *Le Précurseur*; the memoir of H.J. Gisquet, prefect of police; and the memoir of Lucien de la Hodde, one of Louis-Philippe's spies. We have already seen the first article, and it does not give any evidence to support Rigatelli's claim beyond the fact that only one of the guns in the duel was loaded. As for Gisquet's memoir, I have found no mention of Galois or his duel in it. Hodde's memoir mentions Galois once:

> A meeting of the leading members took place on May 7, in the suburb of Saint-Martin, and the principle of insurrection, already generally agreed upon among the subordinates, was voted on in an official matter.
>
> It just so happened that, a few days after, an influential republican, M. Gallois, was killed in a duel; his duel would be the pretext for a taking of arms [17, 86].

Perhaps Rigatelli extrapollated the procedings of the meeting from this last sentence, but this sentence merely reaffirms Weill's claim that an uprising was planned for Galois's funeral and no more.

Some details of Rigatelli's account are almost certainly fabricated. She describes for example Galois's excitement as he prepares to volunteer his life for the movement and even has him explaining that "his life had become pointless. All that was left for him was to offer it to the only thing he still loved: France. The corpse they needed would be his." [31, 109]. Yet I could not find Galois or anyone else giving this explanation in any primary source. Rigatelli recognizes that this wish for death contradicts the sentiments that Galois expresses in his letter to Chevalier. In an attempt to resolve this, she claims that Galois promised to visit Chevalier in his letter "so as not to arouse suspicions in his friend's mind" [31, 110]. Rigatelli must also contend with Galois's two letters to his republican friends on the night before his duel, in which he laments that he will lose his life over a woman rather than for France. Of these, she remarks that "the letters are so skillfully written that they have given rise to different versions of the events in various biographies of Galois" and asserts that these letters were merely to "prevent anyone from suspecting the true circumstances of [Galois's] death" [31, 110]. These claims seem less like fact and more like Rigatelli's attempt to dismiss contradictory evidence.

Rigatelli asserts further that, after Galois's death, the *Société des Amis du Peuple* "would only have the task of spreading the news that the duel was actually a police ambush" [31, 109]. If we recall the first letter that Galois addressed to his fellow republicans (to the large group rather than to his two friends), he immediately admits that he will die in a *cancan*, a frivolous quarrel, rather than for his country. In Rigatelli's version of events the Société doctors these letters to make the duel plausible. It's hard to imagine someone who did not already have a particular story in mind reading these letters and suspecting a police set-up. If Galois himself sought to frame the duel as a trap by the police, why would he not have written so himself? Why would he say his opponents are "of good faith" if he was actually the victim of police spies? Neither this letter nor the letter Galois sent to his two other republican friends paints him as a heroic figure over whose death the people of Paris would go to war.

Rigatelli also points out, more plausibly, that Galois repeated with certainty in his letters that he will die in the duel, suggesting that his death was planned. Chevalier explicitly stated he obtained the letters: they were found lying on Galois's table in his room, and Galois's mother brought them to him. In other words, the existence of these letters does not show that Galois was certain of his death. Moreover, the fact that he left them in his room suggests, on the contrary, the he was *uncertain* of his death. Galois did not plan to die in the duel.

No one has convincingly attributed a political aim to the duel. Beyond perhaps a suicidal commitment to honor that was widely shared in Europe in the early nineteenth century, we also have no evidence that Galois provoked the duel in an act of suicide. Galois's own writings contradict these theories. Further study may bear some fruit. It would be helpful, for instance, to read Chevalier's side of his correspondence with Galois, in which he presumably counsels Galois to seek revenge for injustices against him. Perhaps further clues lie in d'Herbinville's own correspondence, almost certainly long lost. Though Galois may have only attempted reconciliation in person, these attempts may have left a paper trail as well. We still do not even know who his second opponent was. Nevertheless, we can attempt to draw some conclusions. D'Herbinville was almost certainly Galois's killer, and the duel was most likely not a political act or a form of suicide. Galois himself, though he provides few other details, tells us what the duel actually was: a quarrel over a woman.

5 Galois's mathematics

In his Testamentary Letter, Galois wrote that "after this there will be, I hope, people who profit from deciphering all this mess" [5, 185]. The mess which mathematicians deciphered in the decades after Galois's death did not limit itself to the contents of the Testamentary Letter but rather consisted of his entire opus. One of the fundamental mathematical objects in modern algebra, for which Galois provided the first formal definition, is the *group*. Many mathematicians consider Galois to be the founder of group theory, and hence one of the founders of modern abstract algebra. Galois's important work extends beyond solving polynomial equations to include elliptic curves and finite fields. I will focus here on his famous necessary and sufficient condition for solving polynomial equations, the subject of his 1831 *Premier Mémoire*. The rich theory which surrounds this result and which has much of its origin in Galois's work is today known as Galois Theory.

5.1 Polynomial equations

A polynomial is a function of the form

$$f(x) = a_0 + a_1 x + \cdots + a_{n-1}x^{n-1} + a_n x^n \,,$$

where the coefficients $a_0, \ldots, a_n$ are rational numbers (i.e., fractional numbers as opposed to numbers such as $\sqrt{2}$ or π). The exponent n of the last term is called the *degree* of $f(x)$. A *root* of a polynomial is a value α such that $f(\alpha) = 0$. To "solve" a polynomial means to find its roots. Most high school students learn an equation for solving polynomials of degree 2, called *quadratic* polynomials:

$$f(x) = ax^2 + bx + c \qquad \Rightarrow \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \,.$$

The Babylonians knew how to solve quadratics, and formulas for solving polynomials of degree 3 and 4 (*cubic* and *quartic* polynomials respectively) were found around 1540 [22, 18]. We call such methods "solving the polynomial by radicals" because they only involve taking roots of numbers (square roots, cube roots, etc.; not to be confused with the roots of a polynomial!) and the elementary arithmetic operations $+$, $-$, $\times$, and $\div$. Among the works that Galois devoured at *Louis-le-Grand* was Lagrange's famous *Réflexions sur la résolution algébrique des équations*. This work, published in 1770, laid out the known methods for cubics and quartics and informed much of the work in solving polynomial equations in Galois's time.

Lagrange's general tactic was to reduce these equations to auxiliary *resolvent equations* that were one degree lower than the original polynomials. Lagrange derived these resolvent equations by taking rational functions of the roots $x_1, \ldots, x_n$ of the polynomial (i.e. expressions combining the roots with $+$, $-$, $\times$, and $\div$) and then examining what different values these rational functions took upon permuting $x_1, \ldots, x_n$. For example, forming the expression $x_1x_2 + x_3x_4$ from the roots of a quartic polynomial and permuting the indices (e.g. $x_1x_3 + x_2x_4$) in all 24 different ways yields three distinct values, y_1, y_2, y_3. Lagrange then analyzed the resulting cubic polynomial $(x - y_1)(x - y_2)(x - y_3)$

to determine the roots of the original quartic polynomial [22, 19].

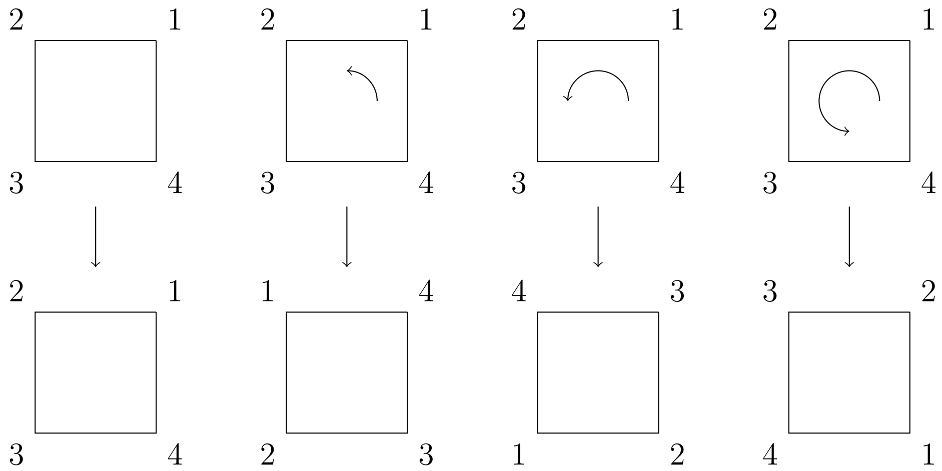
Galois later used Lagrange's idea of permuting the roots in his own analysis of polynomials. When Lagrange attempted this method with the polynomial of degree 5 (the *quintic*), the resolvent equation had a higher degree of 6, and he could proceed no further. In 1824, the mathematician Niels Henrik Abel proved that no such method exists. In other words, Abel showed that, for degrees greater than 4, there are polynomials that cannot be solved by radicals. Studying the permutations of roots, Galois went further than this and gave a necessary and sufficient condition for determining whether *any* polynomial was solvable by radicals. His work reaffirmed that any polynomial of degree 4 or lower can always be solved by radicals and laid the groundwork for solving certain polynomials of higher degree. Galois's methods, as well as his notion of a group, ushered in the era of modern algebra.

5.2 Groups

A *group* is a collection of elements (e.g. numbers, transformations, etc.) together with an operation that takes two elements of the group and outputs another element. The integers are an example, with the operation of $+$: for any integers x and y, we have the single number $x + y$. We call this property *closure*: the group is "closed" under its operation. Galois coined the term "group" for such an object, and took closure as the sole defining property of a group: "if in such a group one has substitutions S and T, one is sure to have the substitution ST" [26, 115]. Today, we enlarge the definition to include the existence of an *identity* element such that combining the identity element with any other element x just yields x. In the integers, the identity is 0: $a + 0 = a$. This modern definition also includes the existence of an *inverse* for every element: for any x, there is an inverse $-x$ which may be combined with x to obtain the identity.

We will consider here two examples of a group. First, think of a square with its four vertices labelled from 1 to 4 counter-clockwise. Next, imagine rotating it in increments of 90°. Rotating by 360° is the same as not rotating at all, and any rotation after that is the same as rotating by one of the angles 90°, 180°, or 270°, so we have four rotations.

The identity rotation is the 0° rotation. We compose rotations by doing one after the other, so 270° followed by 180° gets 90°. Composing any rotation with 0° gets the same rotation. Figure 1 shows each of these rotations. Notice how each of these rotations gives a new ordering of, or *permutes*, the vertices.



(a) 0° (b) 90° (c) 180° (d) 270°

Figure 1: The group rotations of a square, C_4

We write the set of rotations, which we call C_4, as $\{0^\circ, 90^\circ, 180^\circ, 270^\circ\}$. Each of these rotations can be obtained by successively applying 90°. If we denote this by r, then we may write every element in terms of r. 270°, for example, which is obtained by doing r three times, is rrr, or r^3. r^4 gets 0°, the identity element. The group of these rotations is then $C_4 = \{1, r, r^2, r^3\}$. In this way r *generates* this group, and we can write it as $\langle r \rangle$ to indicate that we get each element by applying r. The group C_4 is an example of a class of groups called *cyclic groups*, which are groups that are generated by a single element. Here, for example, C_4 is generated by the single element r.

For the next example, think of an equilateral triangle, and label the vertices 1, 2, and 3 counterclockwise. We now have three rotations, and like before we can get these by applying 120°, which we now call r here. There is also another symmetry of the triangle, that of reflecting over an axis through one vertex to the midpoint of the opposite side (see Figure 2).

Let s be one of these reflections. When we apply s, we permute the vertices, this time leaving one of them fixed and swapping the other two. If we do s first and then r, we

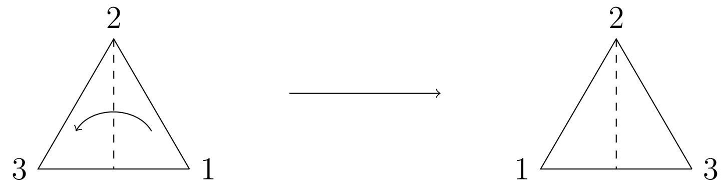


Figure 2: Reflection, s

get the symmetry rs. If we do them in a different order, we get a different symmetry, sr (which we could in fact write as r^2s). There turn out to be 6 symmetries in total, and we call this group D_6:

$$D_6 = \{1, r, r^2, s, rs, r^2s\}.$$

Though one can see group theory implicit in the works of Lagrange, Gauss, and Abel (e.g., Lagrange's permutations of roots), Galois made this concept explicit. He was the first to give a formal definition of a group, though in his definition he specifies only that the group is closed under its operation. Galois also treated groups solely as *groups of permutations*, i.e., groups whose elements were permutations of some number of objects. The group D_6 is not the symmetries of the triangle but rather all the different ways you can permute the three vertices $1, 2$, and 3. He did not view the reflection s, for example, as a reflection, but rather a permutation which swaps 1 and 3. As for C_4, it consists of some permutations of the vertices $1, 2, 3$, and 4, but not all of them: in particular, it consists of the permutation which cycles 1, 2, 3, and 4, i.e., $\{1, 2, 3, 4\} \to \{4, 1, 2, 3\}$, each cyclic permutation corresponding to a rotation of the square. (Galois was likely aware of these cyclic permutations, and in the examples where he worked with them he called them "circular substitutions." It was Cauchy, however, who provided the current definition for a cyclic substitution [26, 27].) As Neumann points out [26, 22], Galois's treatment of groups as permutation groups helps explain why he considered only the closure property of groups as the essential property without considering the existence of an identity element or an inverse. In a finite group, being closed under multiplication (the most important property for Galois) actually implies the existence of an identity as well as an inverse for every element.

Within any group we have *subgroups*, which are smaller collections of the elements that are themselves groups. For example, if we take r in D_6 and apply it to itself, we get $H = \{1, r, r^2\}$. Combining any two elements in this set gets us another element, since it will yield r^k for some $k = 1, 2, 3, \ldots$. The group D_6 contains all the elements of H, which is why we call H a *sub*group of D_6, written symbolically as $H \subseteq D_6$. The set $\{1, r, s\}$ is not a subgroup, on the other hand, because we can make rs with r and s, but rs falls outside the set. One common way to create subgroups is to take some elements from the group and then "close" them under the group operations. In our example with $\{1, r, r^2\} = \langle r \rangle$, we apply r to itself until we get the identity element. Since reflecting twice in the same axis yields the identity, the reflection s generates the subgroup $\langle s \rangle = \{1, s\}$. The element rs generates $\langle rs \rangle = \{1, rs\}$. (It turns out $sr = r^2 s$, so $(rs)(rs) = r(sr)s = r(r^2 s)(s) = (r^3)(s^2) = 1$.) We can arrange the subgroups of D_6 in a *subgroup lattice*:

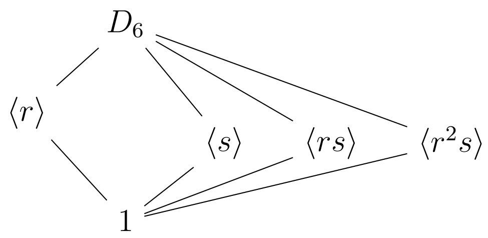


Figure 3: Subgroup lattice of D_6

The subgroups higher up have more elements than those below. The lines indicate containment: D_6 contains all its subgroups, and all subgroups contain 1, but here none of the subgroups contain each other. Subgroup lattices play a key role in the Fundamental Theorem, though Galois himself did write of subgroups as being in a lattice.

Galois not only recognized what subgroups were (though he referred to them as "groups" rather than "subgroups" [26, 85]), he also identified a special type of subgroup. Suppose we took our subgroup $\{1, r, r^2\}$ and applied r to every element on the left. We would get exactly the same set, just in the order $\{r, r^2, 1\}$. If we applied s to every element, we would get three different elements: $\{s, sr, sr^2\}$. Notationally, we represent the first case as $rH(= H)$ and sH. Such things are called *cosets*. (The coset sH is not a group, though Galois called it a group.) Imagine that we instead apply r and s on the right so that we get Hr and Hs (remember that the order of the elements in the set doesn't matter here):

$$\begin{aligned} rH = \{r, r^2, 1\} \quad &= \quad \{r, r^2, 1\} = Hr \\ sH = \{s, sr, sr^2\} \quad &\quad \{s, rs, r^2s\} = Hs \\ = \{s, r^2s, rs\} &= \{s, rs, r^2s\} = Hs\,. \end{aligned}$$

The left column consists of *left cosets* and the right column consists of *right cosets*, and we see here that they are the same for H. Galois called such groups "proper" [26, 85] in his Testamentary Letter. Contrast this with the subgroup $\langle s\rangle = \{1, s\}$, which we'll call K:

$$rK = \{r, rs\} \neq \{r, sr\} = Kr\,,$$

since $rs \neq sr$. In H, the left and right cosets were always equal, whereas with K, we've found a pair of left and right cosets that do not coincide. We have just seen that H is a normal subgroup of D_6, whereas K is not. Galois called the former type of subgroup a *proper* subgroup, though today we call such groups *normal subgroups.* Galois recognized that the cosets of a subgroup partition a group into sets of the same size (so H in the example above partitions D_6 into two cosets of 3 elements) and that, when the subgroup is proper, the cosets *themselves* form a group. Normal subgroups not only play an important role in the Fundamental Theorem, but also in all of the group theory that developed after Galois.

One difference between Galois's group theory and the theory today is the use of "set notation" as we used with $rK = \{r, rs\}$ in the equation above. The mathematician Georg Cantor developed set theory in 1870, so Galois did not have the convenience it offered. In describing proper subgroups to Chevalier in the Testamentary Letter, for example, Galois wrote:

> ...When a group G contains another H, the group G can be partitioned into groups each of which is obtained by operating on the permutations of H with one and the same substitution, so that $G = H + HS + HS' + \cdots$. And also it can be decomposed into groups all of which have the same substitutions, so that $G = H + TH + T'H + \cdots$. These two kinds of decomposition do

> not ordinarily coincide. When they coincide the decomposition is said to be proper. [26, 85]

In set theoretic notation, we could say that, for a given g in the group, $gH = \{gh \,|\, h \in H\}$ and $Hg = \{hg, \,|\, h \in H\}$. A subgroup is normal if $gH = Hg$ for all g in the group. This simplification is analogous to that brought about by the introduction of algebraic notation. Rather than stating the Pythagorean Theorem as "in a right triangle, the squares of the lengths of the two legs equals the square of the length of the hypotenuse," we may simply say $a^2 + b^2 = c^2$.

5.3 Fields

A different type of structure in algebra is a *field*. Like groups, a field is a set of elements, but with these elements we now allow two operations: addition and multiplication (and with them subtraction and division). The most familiar field, and one of the main ones with which Galois worked, is the field of rational numbers, written $\mathbb{Q}$. This is the set of fractions, where the numerator and denominator are both integers (though the denominator can't be 0). The numbers $\frac{1}{2}$ and $\frac{17}{20}$ are examples, but $\mathbb{Q}$ also includes integers such as $2 = \left(\frac{2}{1}\right)$. Galois and many mathematicians before him considered the question of how to find the roots of a polynomial with coefficients in $\mathbb{Q}$. The polynomial $x^2 - 1$ has the roots ± 1, for example.

Since ancient times mathematicians have known that not all numbers are rational. The square root of 2, written $\sqrt{2}$ is a famous example; there is no fraction or ratio of integers, which, when squared, produces 2. Thus, the polynomial $x^2 - 2$ has no roots in $\mathbb{Q}$ since its two roots, $\pm\sqrt{2}$ are not rational. In Galois's time, mathematicians classified such roots as not "rationally known."

To adjoin a root to $\mathbb{Q}$, we can ask ourselves what a field which contains $\mathbb{Q}$ and $\sqrt{2}$ would need. Just as with groups, when we combine two elements (with any of the four operations), we need to get another element of our field. Thus, $1 + \sqrt{2}$, $2\sqrt{2}$, and $\frac{1}{\sqrt{2}}$ all need to be in our field. Adjoining $\sqrt{2}$ to $\mathbb{Q}$ entails adding on all these other elements as well, as we must "close" the set under the four operations to get a field. Since $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$, we're most concerned with being able to add and multiply by $\sqrt{2}$. Ultimately, our field is the set of all numbers of the form $a + b\sqrt{2}$, where a and b are rational numbers. We may write this as

$$\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}.$$

Since two rational numbers are needed to describe an element of this field, we say today that $\mathbb{Q}(\sqrt{2})$ is a degree 2 extension of $\mathbb{Q}$. Galois and his contemporaries referred to such elements as "rational functions of $\sqrt{2}$" since they come from applying the four basic operations to $\sqrt{2}$. In this new field, $\sqrt{2}$ is "rationally known." In the "Principles" section that starts his *Premier Mémoire*, Galois wrote:

> ...one could agree to regard as rational every rational function of a certain number of determined quantities, supposed known a priori. For example, one could choose a certain root of a whole number, and regard as rational every rational function of this radical.
>
> When we thus agree to regard certain quantities as known, we shall say that we *adjoin* them to the equation which it is required to solve. We shall say that these quantities are *adjoined* to the equation.
>
> That having been said, we shall call *rational* every quantity which can be expressed as a rational function of the coefficients of the equation together with a certain number of quantities *adjoined* to the equation and agreed arbitrarily.
>
> One sees moreover that the properties and the difficulties of an equation can be quite different according to the quantities which are adjoined to it. For example, the adjunction of a quantity can render an irreducible equation reducible. [26, 109]

Thus, in $\mathbb{Q}$, the polynomial $x^2 - 2$ does not "reduce" into the product of two lower-degree polynomials, whereas in $\mathbb{Q}(\sqrt{2})$ it reduces into $(x - \sqrt{2})(x + \sqrt{2})$. Note that Galois would have called $\sqrt{2}$ "rational" in the larger field.

If we wanted a field that contained all the roots of $(x^2 - 2)(x^2 - 3)$, however we'd have to work a bit harder. This polynomial also has the roots $\pm\sqrt{2}, \pm\sqrt{3}$. $\mathbb{Q}(\sqrt{2})$ does not contain $\sqrt{3}$, so we must *adjoin* $\sqrt{3}$ to this field. We call this field $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ for convenience. As before, we could think of this new field as the set of of the form $a + b\sqrt{3}$, where now a, b lie in $\mathbb{Q}(\sqrt{2})$ rather than in $\mathbb{Q}$. To have the coefficients be rational numbers, however, we must go a bit further. The elements $a + b\sqrt{2} + c\sqrt{3}$ certainly lie in our field (where a, b, and c are rational numbers), but we can now multiply together $\sqrt{2}$ and $\sqrt{3}$

to get $\sqrt{6}$. Therefore, our set is

$$\mathbb{Q}(\sqrt{2}, \sqrt{3}) = \{a + b\sqrt{2} + c\sqrt{3} + d\sqrt{6} \,|\, a, b, c, d \in \mathbb{Q}\}\,.$$

Since we now need four rational numbers to describe an element in this field, it is a degree 4 extension of $\mathbb{Q}$. In the parlance of Galois's time, these are the rational functions of $\sqrt{2}$ and $\sqrt{3}$. Observe that this field includes all the elements of $\mathbb{Q}(\sqrt{2})$, as well as those of $\mathbb{Q}(\sqrt{3})$ and $\mathbb{Q}(\sqrt{6})$ (which we could construct from $\mathbb{Q}$ just as we did with $\mathbb{Q}(\sqrt{2})$). These smaller fields of $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ are called *subfields*, and they are analogous to subgroups of groups. Symbolically, we write $\mathbb{Q}(\sqrt{2}) \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$, etc. As with groups, we can display this information in a lattice, called a *subfield lattice*:

$$\begin{tikzcd}
 & \mathbb{Q}(\sqrt{2}, \sqrt{3}) \arrow[dl, dash] \arrow[d, dash] \arrow[dr, dash] & \\
\mathbb{Q}(\sqrt{2}) \arrow[dr, dash] & \mathbb{Q}(\sqrt{6}) \arrow[d, dash] & \mathbb{Q}(\sqrt{3}) \arrow[dl, dash] \\
 & \mathbb{Q} &
\end{tikzcd}$$

Figure 4: Subfield lattice of $\mathbb{Q}(\sqrt{2}, \sqrt{3})$

As with the subgroup lattice, the idea of a subfield lattice was developed after Galois's time.

5.4 The Fundamental Theorem of Galois Theory

In his 1770 paper "Reflections on the solution of algebraic equations," Lagrange considered quantities obtained by permuting the roots of a polynomial [22, 18]. In his "Premiere Mémoire," Galois identified that the permutations of a polynomial's roots form a group, which he called the "group of a polynomial," though today we call it the *Galois group*. The four roots of the polynomial we've been considering, $(x^2 - 2)(x^2 - 3)$, are $\pm\sqrt{2}, \pm\sqrt{3}$. There are 24 permutations of four elements, but not all of these permutations are permissible in the Galois group. These permutations were required to preserve all rational relations between the roots; they also leave the rational numbers unchanged. For example, $\pm\sqrt{2}$ satisfy $\left(\pm\sqrt{2}\right)^2 = 2$. If we were to swap $\sqrt{2}$ and $\sqrt{3}$, we would have the false relation $\left(\sqrt{3}\right)^2 = 2$. In our case here, the only permissible permutations are σ, which swaps $\sqrt{2}$ and $-\sqrt{2}$ while leaving $\sqrt{3}$ fixed, τ, which swaps $\sqrt{3}$ and $-\sqrt{3}$, and $\sigma\tau$, which performs both swaps (we also have the "identity" permutation of doing nothing). Each of these does nothing to rational numbers such as 3, $\frac{1}{3}$, or $\frac{1}{2} + \frac{2}{5}$.

Galois's Proposition I in his *Premier Mémoire* asserts that such a group exists for any polynomial with the property that (i) every quantity that is fixed under the Galois group is "rationally known" (in this case, contained in $\mathbb{Q}$), and that (ii) every quantity in our field that is "rationally known" is fixed. From Proposition I (which Galois himself did not prove) comes the Fundamental Theorem of Galois Theory: for a field K containing all the roots of a polynomial, the subfields of K correspond exactly to the subgroups of the Galois group for that polynomial. More precisely, each subgroup of the Galois group corresponds to the subfield of K fixed by that subgroup. Though this is the main part of the Theorem, it is usually stated with other parts as well. For example, the degree of K over one of its subfields F is equal to the number of elements of the subgroup correspinding to F. Galois himself uses this consequence when considering the case of a 4^{th} degree polynomial whose Galois group is S_4 (the set of 24 permutations of 1, 2, 3, and 4): as he joins square roots to the field, the size of the Galois group is cut in half each time. By this method he demonstrates how to solve a 4^{th} degree polynomial.

Recall that σ swaps $\sqrt{2}$ and $-\sqrt{2}$, leaving $\sqrt{3}$ fixed, that τ swaps $\sqrt{3}$ and $-\sqrt{3}$, leaving $\sqrt{2}$ fixed, and that $\sigma\tau$ swaps both pairs of roots but leaves their product fixed. The fact that σ does nothing to $\sqrt{3}$ means that it leaves $\mathbb{Q}(\sqrt{3})$ fixed. The same is true with τ and $\sigma\tau$ for $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{6})$. The Fundamental Theorem asserts that this is no coincidence and implies that the subfield lattice for $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ (left) and the subgroup lattice for V_4 (right) must have the same structure:

$$
\begin{tikzcd}[arrows=dash]
 & \mathbb{Q}(\sqrt{2},\sqrt{3}) \arrow[dl] \arrow[d] \arrow[dr] & \\
\mathbb{Q}(\sqrt{2}) \arrow[dr] & \mathbb{Q}(\sqrt{6}) \arrow[d] & \mathbb{Q}(\sqrt{3}) \arrow[dl] \\
 & \mathbb{Q} &
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}[arrows=dash]
 & 1 \arrow[dl] \arrow[d] \arrow[dr] & \\
\langle\tau\rangle \arrow[dr] & \langle\sigma\tau\rangle \arrow[d] & \langle\sigma\rangle \arrow[dl] \\
 & V_4 &
\end{tikzcd}
$$

Figure 5: The lattices of $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ and V_4

Note that we have inverted the lattice for V_4 so that the smaller subgroups are higher up to emphasize the correspondence. Reading the two lattices, $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ is fixed only by the identity permutation, 1, $\mathbb{Q}(\sqrt{2})$ is fixed by $\langle\tau\rangle$, etc. The field which is fixed only by the identity permutation is the field which contains all the roots of $(x^2-2)(x^2-3)$, as follows from the Fundamental Theorem.

5.5 Solving by radicals

Consider again $(x^2-2)(x^2-3)$. Adjoining $\sqrt{2}$ to get $\mathbb{Q}(\sqrt{2})$, we have a field which contains some, but not all of the roots. Galois considered not only the Galois group of a polynomial over $\mathbb{Q}$, but also of a polynomial over larger fields, such as $\mathbb{Q}(\sqrt{2})$. Now, this Galois group is the set of permutations which preserve all the algebraic relations in the larger field, $\mathbb{Q}(\sqrt{2})$. There are only two such permutations: the identity and τ, which swaps $\pm\sqrt{3}$; the permutation σ is no longer included since it does not fix all the elements of $\mathbb{Q}(\sqrt{2})$ (in particular it doesn't fix $\sqrt{2}$). The new Galois group, taken over $\mathbb{Q}(\sqrt{2})$, is now smaller. Galois asserts this in his *Proposition II* of the *Premier Mémoire*: "If one adjoins to a given equation the root r...one of two things will happen: either the group of the equation will not be changed, or it will be partitioned into p groups..." [26, 119]. In our example above, if H is the subgroup which fixes $\sqrt{2}$ and G is the whole Galois group, then adjoining $\sqrt{2}$ partitions G into the cosets H and τH. (The new Galois group is in fact the *quotient group* of G by H, which is the set of left cosets of H and exists only when H is normal in G. Galois did not define quotient groups, however, and seemed to take for granted that the subgroup H would be normal.)

The polynomial $(x^2-2)(x^2-3)$ is *solvable by radicals* because its roots can be expressed as (in this case, very simple) rational functions of $\sqrt{2}$ and $\sqrt{3}$. The polynomial x^2-4x-1, for example, is solvable by radicals as well: by the Quadratic Formula, its roots are

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)} = 2 \pm \sqrt{5}\,,$$

which lie in the field $\mathbb{Q}(\sqrt{5})$. Galois's idea was this: a polynomial is solvable by radicals if and only if we can reduce its Galois group to the identity permutation by adjoining radicals such as $\sqrt{2}$ and $\sqrt[3]{7}$. On the one hand, if the polynomial is solvable by radicals, then "it will always be the case that after a certain *finite* number of root extractions [adjunctions] the group will have to become smaller, otherwise the equation will not be soluble" [26, 123]. Galois then stated the converse, as well as describing explicitly the structure the Galois group must satisfy so that it may be reduced to the identity by adjoining radicals, though this condition is a bit too technical for this paper. This condition, however, is today called "solvable" because of its relevance to determining whether or not a polynomial is solvable by radicals. In other words, a polynomial is solvable by radicals if and only if its Galois group is solvable, and Galois defined what structure a group had to satisfy in order to be solvable.

By analyzing the Galois group for any polynomial, we can determine whether or not that particular polynomial is solvable by radicals. Since the Galois group of a degree n polynomial acts as a set of permutations on n objects (these n objects being the roots), it must be a subgroup of the group S_n. For $n \leq 4$, S_n is solvable, so the polynomials of degree 2, 3, and 4 can always be solved by radicals. Galois showed the solvability of S_4 in his *Premier Mémoire* [26, 125]. The reason why polynomials of higher degree cannot always be solved by radicals is simple: the group S_n is not solvable for $n \geq 5$, and there are always degree n polynomials whose Galois groups are S_n. These polynomials cannot be solved by radicals.

5.6 Galois's reception

Galois's contemporaries were not convinced. Part of this is due to the novelty of his work and methods. Lagrange's work on solving polynomial equations meticulously detailed how to find the roots of the polynomials, whereas Galois said little on how to determine the Galois group of a polynomial in order to even determine whether the equation was solvable. In the illustrative example we worked with, we could easily see the roots of the equation from the start, which seems to defeat the point of considering the Galois group at all. How could one find the Galois group of, say, $x^5 - x - 1$? In sorting through

Galois's mess, mathematicians only developped methods for computing the Galois group from the polynomial's coefficients decades later. On this issue, Galois wrote:

> If you now give me an equation that you have chosen at will, and you wish to know whether or not it is soluble by radicals, I will have nothing to do other than to indicate to you the way to respond to your question, without wishing to charge either myself or anyone else with doing it. In a word, the calculations are impractical. [26, 227]

This viewpoint went against the mainstream emphasis on actually determining the roots through calculation. Most textbooks of the time, such as those of Lacroix and Legendre, and research on the topic focused on computing the roots. From this perspective, Galois's work was severely lacking. To make matters worse, Galois's proofs were concise to a fault. In his Lemma I in which he proves an irreducible equation divides any equation with which it has a root in common, Galois argues: "For the greatest common divisor of the irreducible equation and the other equation will again be rational; therefore, etc." Galois himself defended himself on this matter (or rather, he railed against what he saw as an excessive manner of mathematical argument) in his *Préface*, written in December of 1831. Arguments such as this explain why Poisson failed to comprehend Galois's work.

Mathematicians did not know of Galois's work immediately after his death. Mathematicians only began to develop Galois's ideas after the mathematician Joseph Liouville published Galois's works years later.

5.7 Liouville and Galois's vindication

Galois's work appeared in Liouville's *Journal de Mathématiques pures et appliqueés* in 1846. Liouville had gotten the manuscripts Chevalier and Galois's brother, Alfred [5, IX]. His journal was internationally known, and soon mathematicians across the continent began sorting through the mess that Galois left. In publishing Galois's work, Liouville wanted to do more than expand the field of mathematics. As the historian Caroline Ehrhardt argues, Liouville sought to win a quarrel with another mathematician, Guillaume Libri, and bolster his own reputation as a competent mathematician.

On August 21 1843, Liouville questioned the validity of a well-received theorem by

Libri on the solvability of equations. Among Libri's replies was that Liouville was not regarded as an expert in the material. In response to this, Liouville annonced his intention to publish Galois's papers and to give his own commentary [24, 548–549]; [11, 547]. The purpose of this was twofold. Firstly, by succeeding where Poisson and Lacroix had failed, Liouville would prove himself as an algebraist. Secondly, in legitimizing Galois's work Liouville would disprove Libri's "theorem," as Galois had argued contrary to what Libri asserted [25, 553]; [11, 548].

When Liouville finally published Galois's work three years later, he prefaced it with a short commentary on Galois's life, which is relevant to understanding the Galois legend. In his view, Galois spent his last years "fruitlessly, in political unrest, surrounded by clubs or behind the bars of Sainte-Pélagie." On the duel, he attributes it "without doubt to some frivolous quarrel" [23, 381] (which is consistent with what I have argued about the duel). Rather than heap opprobrium on Poisson and Lacroix for refusing to publish Galois's work, he deems their protests well-founded. He quotes Descartes in saying that "on transcendental questions, be transcendentally clear," and remarked that "Galois too often neglected this precept, and we understand that these illustrious geometers judged it best to put him on the right path by the severity of their sage counsel, a beginner full of genius, but inexperienced' [23, 382].'

After briefly addressing this awkward topic, Liouville concludes "but now everything has changed. Galois is no more! Let us guard ourselves against pursuing useless critiques; let us leave the defeats and see the qualities" [23, 382]. Liouville realized the risk of increasing his reputation in the mathematical establishment with the work of someone who detested and openly railed against that very establishment. This discussion of Galois's life, as Ehrhardt observes, addresses this issue. It absolves Poisson and Lacroix of any slight on their part, reversing the narrative so that they in fact sought to *help* Galois. Liouville also disparages the more controversial aspects of Galois's character so that his readers may focus exclusively on the mathematics. Liouville's version of Galois's life starkly contrasts with that of Bell in this way, but we will see later that many mathematicians took Liouville's stance on Poisson and Lacroix's rejection of Galois.

Liouville ultimately never published his commentary on Galois's work, but he wrote in his preface that

> My zeal was soon rewarded, and I enjoyed great pleasure the moment when, after filling in some slight holes, I recognized the entire exactitude of the method by which Galois proved, in particular, this beautiful theorem: [...] This method, truly worthy of geometers' attention, suffices to ensure our compatriot a place among the small number of *savants* who have earned the title of inventor. [23, 383]

Enrico Betti, a year after asking Liouville to publish this commentary, published his own in 1852, giving the first public commentary on Galois's work. Joseph Serret gave the first text-book account of Galois theory in 1866 [27, 340]. Perhaps the most important work was the mathematician Camille Jordan's *Traité des substitutions et des équations algébrique*, a several hundred page treatise that made great strides in formalizing Galois's work. Despite its length, Jordan wrote that it was "only a commentary" on Galois's work [20, VIII].

6 The Galois legend

6.1 Bell's Galois

Over the years since Galois's death, his story has become a legend. Eric Temple Bell's emotionally charged telling in *Men of Mathematics* is the most influential version. Mathematicians and scientists such as John Nash, Andrew Wiles, and Freeman Dyson cite E.T. Bell's account as an inspiration for their own careers. Bertrand Russell, the famous philosopher, logician, and mathematician, said (in a quote on the back of *Men of Mathematics*)

> Professor Bell has done his work well.... Any [one] engaged in learning mathematics will profit by reading him, since he humanizes the subject and helps to a realization of the historical environment. [2]

I choose Bell's account as a starting point in analyzing the Galois legend, because it is the perfect telling of the legend. When mathematicians imagine the young man Galois, they imagine Bell's Galois.

Bell titles his chapter on Galois "Genius and Stupidity," and writes that "in all the history of science there is no completer example of the triumph of crass stupidity over untamable genius than is afforded by the all too brief life of Évariste Galois" [2, 362]. This sentence alone characterizes almost completely the Galois legend: he was a fiery genius, he was rejected by idiots, and he died young. Bell turns Galois's world against him from the beginning. Louis-le-Grand was a "dismal horror...dominated by a provisor who was more of a political gaoler than a teacher." The "tyranny" Galois witnessed at Louis-le-Grand "warped one side of his character for life." In this stifling environment Galois's own genius came on him "like an explosion" [2, 363], and he read through Legendre's work "as easily as other boys read a pirate yarn" [2, 364], after which he read Lagrange and Abel. At the age of sixteen Galois failed the entrance examinations to the prestigious *École Polytechnique*, and "he was not alone in believing his failure the result of a stupid injustice." Thankfully, Galois met someone able to tame his genius and transform him into "the Abel of France": Louis-Paul-Émile Richard, a humble, excellent man who put his own students before himself [2, 367].

Galois began to publish his work, eventually submitting it to the Academy. The prominent *Académie* mathematician Cauchy promised to present Galois's work, but, according to Bell, he forgot to do so and lost Galois's memoir, "fann[ing] the thwarted boy's sullen contempt of academies and academicians into a fierce hate against the whole of the stuid society in which he was condemned to live" [2, 369]. Soon after this, Galois failed the entrance exam to the *École Polytechnique* for the second and final time under the judgment of "men who were not worthy to sharpen his pencils." During the exam, as Galois began to realize he would fail, Bell recounts that Galois threw his eraser at "his tormentor's" face [2, 369].

In February of 1830, Galois composed three papers for the Academy of Sciences, which was holding a competition for the Grand Prize in Mathematics. Though this memoir was worthy of the prize, the man who had been assigned to read it died before he could look at it, and the memoir was lost [2, 370–371]. After this, "his hatred grew, and he flung himself into politics" [2, 371]. After being expelled from the *École Normale*, Galois joined the National Guard. In "one last desperate effort to gain recognition, encouraged by Poisson" [2, 372], Galois submitted a memoir on what is now called "Galois Theory" to the Academy. Poisson responded with a "perfunctory" report, stating the he could not understand it, "but he did not state how long it had taken him to reach his remarkable conclusion" [2, 372]. It was at this point that Galois devoted himself completely to politics. Over the following months, Galois was twice arrested.

A month before Galois was to be released, he was moved to a sanitarium due to a cholera epidemic in March of 1832. It was there that he met "some worthless girl ('*quelque coquette de bas étage*')", and his month-long experience with love disillusioned him [2, 374]. Shortly thereafter, on May 29, Galois announced that he would die in a duel. What happened "is not definitely known," but "extracts from two letters suggest what is usually accepted as the truth: Galois had run foul of political enemies immediately after his release" [2, 375]. The night before the duel, as Bell writes,

> he had spent the fleeting hours feverishly dashing off his scientific last will and testament, writing against time to glean a few of the great things in his teeming mind before the death which he foresaw could overtake him. Time after time he broke off to scribble in the margin 'I have not time; I have not time,' and passed on to the next frantically scrawled outline. What he wrote in those desperate last hours before the dawn will keep generations of mathematicians busy for hundreds of years. He had found, once and for all, the true solution of a riddle which had tormented mathematicians for centuries: under what conditions can an equation be solved? [2, 375]

Bell fabricates many details in his colorful, compelling tale. Stéphanie Dumotel was not "some worthless girl," Dupuy mentions the eraser incident as tradition rather than fact [9, 211], and Galois did not die at the hand of political enemies. He did not frantically put his theory on the solvability of equations to paper the night before the duel but rather developed it over the course of two to three years and multiple *Mémoires*. Rothman debunks many of the myths propagated by Bell in his "The Fictionalization of Évariste Galois." In this section we consider the more enduring facet of this legend: that, despite multiple attempts, Galois was undeservedly rejected by the mathematical community. I will focus in particular on the examiners of the *École Polytechnique*, on Cauchy "losing" Galois's work in 1830, and on Poisson rejecting Galois's work in 1831.

6.2 Galois the persecuted

As we saw in the previous chapter, Liouville admitted that Poisson and Lacroix were right in rejecting Galois's work because Galois gave "proofs" that were often too terse to understand. Many mathematicians, aside from Bell, have agreed with Liouville. In the preface to his treatise on Galois Theory, Jordan wrote that

> in the haste of his writing, [Galois] had left without sufficient proof several fundamental propositions. This hole was soon filled by M. Betti, in an important memoire where the complete series of Galois's theorems was for the first time rigorously established. [20, VI]

Joseph Bertrand, a French mathematician and a contemporary of Liouville, said in an article commemorating Galois that

> In declaring that, despite all his efforts, he did not succeed in understanding, Poisson was obviously sincere; and the reading of the memoire, twice printed since then, gives sufficient explanation...Poisson refused to approve the proof, but he did not condemn it. In good justice, he is irreproachable; he did what he could have done and what he should have done. [3, 341]

As an aspiring mathematician who has read Galois's *Mémoire*, reading it was sufficient explanation for me as to why Poisson rejected it. Bertrand also recalls discussing the proof with Liouville after the latter had published Galois's work:

> Liouville, in publishing, fifteen years after Galois's death, the memoire that Poisson found obscure, announced a commentary that he never gave. I heard him declare the proof very easy to understand. And at the gesture of surprise he saw me make, he added "It suffices to devote onself to it for a month or two, without thinking of anything else." This word explains and justifies the difficulty fairly admitted by Poisson and, without a doubt, recognized by Fourier and by Cauchy. [3, 342]

These three mathematicians are also a small sample of the hundreds of others who have rightfully lionized Galois's theories. Despite the great merit of his ideas, Poisson and Lacroix had reasons to reject Galois's work, which was concise to a fault. Poisson, one of the incompetent villains in Bell's story, recognized these faults but also recognized Galois's merits, as we will see. First, we will trace the origins of the legend by examining what other commentators on Galois's life have said.

In 1921, George Sarton–who effectively founded the discipline of the history of science, its leading journal, *Isis*, and the History of Science Society–wrote in an article of *The Scientific Monthly* about Galois that

> By a strange aberration he did not trouble himself to write his memoirs with sufficient clearness and to give the explanations which were the more necessary because his thoughts were more novel. What a pity that there was no understanding friend to whisper in his ear Descartes' wise admonition: "When you have to deal with transcendent questions, you must be transcendentally clear." Instead of that, Galois enveloped his thought in additional secrecy by his efforts to attain greater conciseness, that coquetry of mathematicians. [34, 369]

Aside from the last sentence, which is a bit reaching, Sarton's point is otherwise consistent with that of mathematicians: Galois should have been clearer. He likely had Liouville's preface in mind when quoting Descartes. As for the entry examinations to the *École Polytechnique*, Sarton remarked that "Galois knew at one and the same time far more and far less than was necessary to enter Polytechnique; his extra knowledge could not compensate for his deficiencies, and examiners will never consider originality with favor." Before leaving Sarton, I will note that with regards to Galois's submission of his first paper to the *Académie*, Sarton claims it was "lost through Cauchy's negligence" [34, 367].

The American mathematician James Pierpont repeats the story about the examiners in a short article he read before the American Mathematical Society in February of 1897. He cites two reasons for Galois's failure: firstly, Galois habitually worked in his head and became embarrassed when asked to present his work on the blackboard before an audience; secondly, "the examinators were flagrantly incapable of appreciating the extraordinary talents of the youth they had before them" [27, 336].

In his *Éloges académiques*, Bertrand rejects this tradition of incompetent examiners. He identifies the examiner of Galois's second entry exam as Dinet, and gives him the following praise:

> Admired for more than twenty years as a mathematics professor, he had had respectable and recognized students: Cauchy, Olinde Rodrigues, Duhamel, Combes and Élie de Beaumont. A man of intelligence besides, [he was] a con-

> scientious and benevolent examiner, and, according to the common opinion, that of all who were the least mistaken [3, 332–333]

According to Bertrand, Dinet always offered simple questions, offering for students a single route “without trap”, counting on his experience and intelligence to judge the candidate’s confidence and the soundness of their arguments. To him, “the manner of proof had more importance than the truths which one proved.” In Bertrand’s version, Galois, after asking Dinet why the examiner had asked him about the arithmetic of logarithms rather than the general theory of logarithms, proceeded to merely say “in a few words what all the candidates knew” [3, 333–334]. Dinet had heard that Galois was a superior student and asked him a more advanced question. According to Bertrand, when Dinet feigned doubt at Galois’s correct answer, Galois “believed to see in this doubt a proof of ignorance” and responded “impertinently” [3, 334]. Though Bertrand goes further than Sarton on this matter, the message is similar: geniuses do not always make for good test-takers. I hesitate to take Bertrand’s account as complete truth, but it matches Galois’s disdain for elaborating “obvious truths” at the expense of brevity.

With regards to Galois failing the entry exam to the *École Polytechnique*, two magazine articles prior to Dupuy helped perpetuate the notion that Galois’s examiners were incompetent. The first is an anonymous article published in the *Magasin Pittoresque* in 1848. This article, like Bell’s account, paints Galois as a persecuted young boy. Of the duel, for example, it stated that Galois was “provoked by men whom he had believed his friends” and that, after the duel, “his witnesses had abandoned him, as well as his adversaries.” Note that in this account of the duel, Galois’s adversaries had provoked Galois rather than the other way around.

The article curiously made no mention of Galois’s troubles with the *Académie*, but it deems Galois’s failure on the *Polytechnique*’s entry exams “a flagrant mistake on the part of the examiners” [12, 228]. Aside from this comment about the entry exam, this article is an important part in the development of the Galois story. Though anonymous, it is likely that the writer had significant contact with Galois’s brother, Alfred. It features a portrait by Alfred, “who from the age of sixteen devoted a veritable *culte* to the memory of his unfortunate brother" (though still religious in tone, the word *culte* in French does not have as negative a connotation as "cult" does in English). The second magazine article, published by the French mathematician Olry Terquem a year later in *Nouvelles annales de mathématiques*, gives a note on Galois at the end of a short biography of his professor, M. Richard. It gives this quote, which Bell repeats [2, 367]: "We repeat here, and we will not cease to repeat a reflection that we have already recorded many a time: A candidate of a superior intelligence is lost on an examiner of an inferior intelligence" [37, 452].

Against Bertrand and Sarton, who attribute Galois's failing the entry exam to his own deficiencies, we have Bell and the two articles from 1848 and 1849. These latter sources base their assessment on the fact that Galois was a genius, but I have not found concrete evidence proving or disproving Dinet's incompetence (Bertrand does not cite a source pertaining to Dinet's performance). We have an abundance of evidence, in Galois's own work and in the testimony of the several mathematicians cited above, that the genius had trouble clearly expressing his ideas. It is thus more plausible that Galois's lack of clarity in argument caused his failure rather than a deficiency on the part of the examiner.

Moving on to the issue of Galois and the *Académie*, the next source we will consider is Auguste Chevalier, Galois's close friend. In the same issue of the *Revue encyclopédique* in which he had published the Testamentary Letter, Chevalier wrote a eulogy for his friend. The beginning paragraphs set the tone for his *Nécrologie*:

> I would despair of sharing with the public the regrets and the veneration with which I surround the memory of my friend, if I did not have the precious deposit of works whose inheritance he entrusted to me.
>
> Galois was known for his ardent republicanism, by the judgements that he suffered and by his long imprisonments: he will be more well-known one day for his scientific genius. *In this respect he was completely ignored* [my italics]; and yet he made numerous attempts with the *Académie des sciences* to spread his discoveries; all his efforts were useless. [7, 744]

Chevalier then describes Galois's first experience in submitting to the *Académie* as well as an issue of precedence between Abel and Galois; essentially, Abel had just published results that formed a significant part of Galois's submission. Chevalier argues that Galois did not know of Abel's work and cites Cauchy as his witness. In the next paragraph, Chevalier ambiguously writes: "this extract [Galois's submission] was lost to its author, who needlessly [uselessly] demanded it from the secretary of the *Académie*: it had been lost" [7, 745]. In mentioning that Cauchy had been assigned to present this memoir in the previous sentence, Chevalier (perhaps unwittingly) suggests that it was Cauchy's fault the memoir had been lost. According to Chevalier, "the lack of attention given by the Institute to the first work submitted to their judgment by Galois started for him the griefs that, until his death, would successively become more and more sharp." As for Galois's submission to the competition for the Grand Prize in Mathematics, his goal was to attract attention to his work. According to Chevalier he did not expect to get the prize, "but he perhaps did not suspect that eighteen years old and the title of student were sufficient grounds to laugh at his pretentions and to condemn without reading his absolutely new ideas" [7, 747], a reading which, while it may capture Galois's own sentiments, is not supported by evidence. He (perhaps rightfully) paints the *Académie* as unsympathetic to losing Galois's memoir:

> "But the loss of this *Mémoire* was a" *very simple thing!* [Chevalier's emphasis] it was with M. Fourier who was supposed to read it, and, "at the death of this *savant*, the *Mémoire* was lost." And the *Académie* carried on regardless... [7, 747]

Chevalier then reveals that Galois resolved from that moment on to abandon his wish to make himself known through the *Académie*, and that "he understood with sadness that this wish was imaginary. And yet he was destined to have a sad experience one more time" [7, 747]. As for Poisson, Chevalier wrote that the *savant* suggested "with kindness" that Galois submit his work. Chevalier then emphasizes, however, that Poisson could not understand the work. Throughout his eulogy of Galois, Chevalier depicts the young mathematician as a misunderstood genius. Though we should be wary of the likely possibility that Chevalier was biased towards his close friend, Chevalier's eulogy faithfully reflects Galois's own feelings. Galois was, after all, the one who started his legend.

6.3 Galois in his own words

In many of his own writings, Galois wrote about injustices committed against him. The most remarkable and famous of these is his *Préface*, which he wrote from Sainte-Pélagie in December of 1831. Here are some extracts from it which reveal plainly Galois's thoughts on the examiners, the *Académie*, and his work:

> Firstly, the second page of this work is not encumbered by the surnames, first names, addresses, honors, and eulogies of some greedy prince whose purse will be opened at the smoke of incense with threat of closing when the censer becomes empty...I say to no-one that I owe to his counsel or to his encouragement all that is good in my work. I do not say it because it would be to lie. If I were to address something to the grandees of the world or to the grandees of science (and at present times the distinction is imperceptible between these two classes of people), I swear it would not be thanks...But I must relate how manuscripts are most often lost in the files of the members of the Institute even though in truth I do not understand such carelessness on the part of men who have on their conscience the death of Abel...It is sufficient to say that my memoir on the theory of equations was deposited in substance with the academy of sciences in the month of February 1830, that extracts from it had been sent there in 1829, that no report followed, and that it was impossible for me to see the manuscripts again...
>
> In the second place, the two memoirs are short and not proportionate to their titles and then there is at least as much French as algebra to such a point that the printer, when one brought the manuscripts to him, believed in good faith that it was an introduction. In this matter I am completely inexcusable; it would have been so easy to review a whole theory from its beginnings, under the pretext of presenting it in a form necessary for the understanding of the work, or perhaps better, without further ado to intersperse a branch of knowledge with two or three new theorems without designating which!...It would have been so easy to transform each sentence ten times, taking care to precede each transformation with the solemn word theorem; or even to arrive by OUR ANALYSIS at results known since the good Euclid; or finally let precede and follow each proposition a formidable procession of particular examples! And of so many ways I did not know to choose a single one!
>
> In the third place, the first memoir is not unsullied by the eye of the master; an extract sent in 1831 to the academy of sciences, was submitted to the inspection of M. Poisson, who came to say in the meeting that he did not understand it. Which to my eyes, fascinated by author's *amour propre* [vanity], proves simply that M. Poisson did not want to or could not understand, but proves certainly in the eyes of the public that my work means nothing.
>
> Thus everything contributes to making me think that in the scientific world, the work I am submitting to the public will be received with the smile of compassion; that the most indulgent will tax me with clumsiness [awkwardness]; and that after some time I would be compared to Wronski or to those

> tireless men who find every year a new solution of squaring the circle. Above all I will endure the mad laughs of Messrs the examiners of the candidates for the *École Polytechnique*, (who, by the way, I am surprised not to see each of them occupying an armchair at the academy of sciences, because their place is certainly not in posterity) and who, having a tendency to monopolize the printing of mathematics books, will not understand without it being formalized that a young man twice scrapped by them also has the pretention to write, not educational books, it is true, but books of doctrine.
>
> All that precedes this, I have said to prove that it is knowingly that I expose myself to be the laughing stock of fools. [5, 3–11]

This passage exemplifies the persecuted Galois. He compared himself to "circle-squarers," who so inundated the *Académie* with proofs of a false theorem (that given a circle one can construct a square with the same area) that the *Académie* stopped accepting proofs for the theorem altogether. (Ironically, mathematics professors today often use the theory of fields that resulted from Galois's work to *disprove* this claim for students.) He also mocks those who require him to provide what he views as excessive justification for his ideas. If M. Richard or Cauchy had encouraged Galois to give clearer arguments, they failed. Needless to say, Galois's portrayal of himself aligns closely with Bell's: a downtrodden genius surrounded by idiots. Many of the stories about Galois's persecution have originated from Galois himself.

6.4 Galois and his contemporaries

Galois presents a different image in the cover letter he sent with his application to the *École Normale*: in it he mentioned that he was "encouraged by people at the top of the learned world to attempt to launch into a mathematical career" (*Archives Nationales, F/17/4176*). In this section I defend Cauchy and Poisson. Cauchy did not lose Galois's paper through negligence, and Poisson was fair in his judgment of Galois's work, if a bit harsh in his delivery. It is plausible that Cauchy both encouraged Galois and convinced him to withdraw from the *Académie*'s judgment the paper which he had supposedly lost through his "negligence." As for Poisson, his critique of Galois's work ends with words of encouragement. We first discuss Cauchy, then Poisson.

The historian René Taton has extensively studied Galois's interactions with the other mathematicians of his era. One article he wrote, "Sur les relations scientifiques d'Augustin Cauchy et d'Évariste Galois" ("On the scientific relations of Augustin Cauchy and Évariste Galois") unravels the tradition of Cauchy bungling the presentation of Galois's work. Based on information in the archives of the *Académie des Sciences*, Taton sets the beginning of Galois's interactions with the *Académie* at May 1829. In fact, at the *Académie* meetings of May 25 and June 1, Cauchy presented two memoirs of Galois's work. We have neither the memoirs Galois submitted nor the report in response, but the *Académie*'s archives indicate that they had to do with polynomials of prime degree. Importantly, the mathematician Niels Henrik Abel, who died in April 1829 at the age of twenty-six, had already published results on this topic, though Galois was not aware of their existence at the time [36, 129].

There's no evidence of further interaction between Cauchy and Galois during 1829. At the beginning of 1830, however, Cauchy agreed to present this same work to the *Académie* again on January 18, 1830. He fell ill beforehand, however, and wrote the following letter:

> Mr. President [of the meeting]
>
> I proposed to present to the *Académie* today: 1° the report on the works of the young Galois; 2° a memoir on the analytic determination of primitive roots, in which I see how one can reduce this determination to the resolution of numerical equations all of whose roots are whole and positive [numbers]. Delayed at home by an slight illness, I regret not being able to attend the meeting today, and I request that you write my name on the agenda for the next meeting for these two objects that I have just indicated.
>
> [Sincerely,] your very humble and obeying servant,
>
> A.-L. Cauchy,
>
> Member of the *Académie*. [36, 134]

The fact that Cauchy had agreed to present Galois's memoir along with his own strongly suggests that Cauchy had seriously considered Galois's work. As Taton points out, this letter directly contradicts the claim that Cauchy simply lost Galois's memoirs. Indeed, Galois himself never claimed that Cauchy had lost his work. Chevalier's eulogy of Galois is the first source that connects Cauchy and with the loss of Galois's work, and Chevalier himself only does this implicitly (and perhaps not deliberately).

Nevertheless, Cauchy did not present Galois's *Mémoire* at the next meeting, on January 25. On this point, Taton has a credible theory: Galois himself allowed this. Indeed, Galois submitted his work to the *Académie*'s *Grand Prix de Mathématiques* the following month, whereas he could hardly expect it to be judged favorably if Cauchy had decided against Galois's wishes not to present his work [36, 136]. Furthermore, in a "Catalogue" Galois wrote of his works over a year later in 1831, he did not mention his work from 1829–the work that, according to legend, Cauchy had lost–and the earliest *Mémoire* he mentioned is the *Premier Mémoire* of January 1830. Taton hypothesizes that Cauchy, recognizing that Galois's work overlapped with that of Abel, persuaded Galois to take the original parts of his work, expand upon them, and submit the result for the *Grand Prix de Mathématiques*. For support, Taton cites an anonymous article which appeared in the newspaper *Le Globe* on June 15, 1831. This article, which argued for Galois's acquittal, wrote that Cauchy had supported Galois:

> It was worthy of the prize, because it solved some difficulties that Lagrange had not been able to resolve. *M. Cauchy on this subject had lavished the author with the greatest praise* [Taton's emphasis]–what did it matter? Someone lost the mémoire, the prize was awarded without the young *savant* being represented in the competition... [16, 668].

Whatever Cauchy's reasons for not presenting the memoir, no evidence exists that he declined to present the memoir against Galois's wishes, and there is certainly evidence that he did *not* carelessly lose it. One question that remains is how did Cauchy react to Galois's submission being lost? Taton is not sure, but notes that Cauchy, who was an ardent conservative on the opposite side of the political spectrum from Galois, left the *Académie des Sciences* in July of that year (one month after the prize was to be announced) right before the July Revolution [36, 142]. Regardless, it is suprising that Cauchy, holding political views so opposed to Galois's, was still willing to help Galois.

Next, we turn to Poisson's rejection. He gave the following response to Galois's *Mémoire* (reproduced in [3]:

> We have made every effort to understand the proof of M. Galois. His arguments are neither clear nor developed enough to allow us to judge their rigor, and we are not even able to give an idea in this report.
>
> The author announces that the proposition featured in his memoir is part of a general theory with many applications. Often it happens that the different

> parts of a theory, in mutually clarifying each other, are easier to understand when taken together than alone. One can therefore wait until the author has published his work in full in order to form a definitive opinion. [3, 341]

This refusal to publish is not a condemnation. Indeed, Poisson invites Galois to flesh out his ideas and submit them again before passing judgment on them. The minutes of the meeting in which Poisson and Lacroix presented the report had a further critique: even if Galois's proposal were correct, he said nothing on how to assess whether or not a given polynomial satisfied Galois's condition (i.e., Galois did not indicate how to construct the Galois group). They argued: "the condition of solvability, if it exists, should be an exterior characteristic that one can verify upon inspection of the coefficients of a given equation, or, at the very most, in resolving other equations of lesser degree than the proposed equation" [29, 660].

Ehrhardt, in her 2010 article "A Social History of the 'Galois Affair' at the Paris Academy of Sciences," interprets this verdict in light of the mathematical milieu in France in 1831. She points out that Lacroix, the other referee on Galois's paper, had dedicated much of his own algebra textbook *Éléments d'algèbre*–with which most young mathematicians were trained–to effectively finding the roots of a given polynomial. This textbook existed with Lagrange's famous *Traité* on the subject, which guided much of the research in algebra during the first thirty years of the 19$^{\text{th}}$ century. Lagrange's work led to a great emphasis on practicality in calculations. In Ehrhardt's words, "the method was only valuable if it ended by an approximate value of the roots." Ehrhardt goes on to cite several reports of *Académie* mathematicians judging submissions according to this criterion [10, 97–98]. Galois's own views directly contradict this principle, as he expounded in his *Préface*. Though Galois's more theoretical approach has led to great advances in mathematics, we should not condemn Poisson and Lacroix for sharing the prevailing view of their time.

To Ehrhardt, the preparation of Lacroix and Poisson's report "highlights Galois's growing status in the scientific field" [10, 108]. First of all, Lacroix and Poisson were well-respected in the *Académie*, and they had a say in awarding prizes and jobs—here Ehrhardt cites to many articles in the *Procès verbaux* in which both draw up list of candidates for various positions at the *Académie* and in the *École Polytechnique* and in which both serve on the committees for the *Grand Prix de Mathématiques.* The *Académie* would not have assigned such men to read the work of a "circle-squarer." Moreover, Lacroix and Poisson wrote reports on fewer than 20 percent of the works to which they were assigned, and the average of papers submitted to the *Académie* which received reports was less than 50 percent. To receive a report at all was a sign of the *Académie*'s interest.

Other mathematicians besides Galois took issue with the *Académie*'s seeming lack of effort in reviewing papers. The mathematician Charles Dupin—who four years later was elected to the *Académie*—complained in a letter to Lacroix in 1814 about receiving no response on a paper he had submitted six months earlier: "Pardon my importunity, but if you knew how cruel it is to see the work you are most fond of abandoned with disdain, almost with contempt!" [10, 111]. Another, Jacques Saigey, claimed in 1829, two years before Galois's *Préface*, that "so few scientists can actually read, even superficially, the work that is submitted to them," and that "favoritism, or mere luck, decided the fate of a paper" [33, 321]. Liouville and Charles Sturm, both of who later became successful mathematicians and who published Galois's works in their respective publications, had submitted several papers to the *Académie* without success. Up-and-coming mathematicians thus turned to extra-academic journals such as the *Bulletin de Férussac* and the *Annales de Gergonne* to share their ideas, and academicians turned to such media as well. Galois's work appeared in both publications. In the *Bullethin de Férussac* of April and June 1829, his work appeared alongside that of Cauchy and Poisson [10, 112].

On the one hand, all of this illustrates a tough, competitive world for any young mathematician. On the other hand, it also shows that Galois was not *persecuted* for his mathematics; as Ehrhardt notes, "his trajectory was very similar to those of the young mathematicians of his generation" [10, 112]. In support of this point, Ehrhardt also notes that mathematicians diversified their research in order to maximize their chances of receiving attention for their work. For Galois's part, he had branched out to working on elliptic functions sometime around late 1830 and early 1831 [5, 516], a topic which Saigey deemed "a field outside of which neither scientific progress nor personal benefit was likely" [10, 114]. Simply put, had Galois had more time to develop his research, he likely would have succeeded in the academic community.

7 Conclusion

Mathematicians fascinated with Galois's life have made him into a tragic hero of mathematics. They often hold him up as a misunderstood genius and a victim of an uncaring scientific establishment. With regards to the latter, the *Académie* was not so uncaring as the story suggests, and two of its prominent mathematicians supported Galois. As for the former, while we cannot contest the brilliance of Galois's ideas, his story is as much an example of how ordinary people mistreat genius as it is a cautionary tale to beginning mathematicians to strive for clear proofs. Galois's failings with the examiners of the *École Polytechnique* were less due to their "incompetence" and more due to the young genius's refusal to answer questions he deemed beneath him.

Other writers have weighed in on the Galois legend and fabricated their own narratives for the duel which ended his life. From the evidence that exists, including Galois's own correspondence, we know instead that he died in a quarrel over a woman. Thankfully, he left his mathematical works intact. The tragedy of Galois's story is not that others failed to recognize his potential. Many of the barriers Galois faced were at least partly of his own creation. Instead, the tragedy is that he died so young when, contrary to his own belief, he had had a promising mathematical career ahead of him. Galois himself must have felt his potential for greatness as he recorded his mathematical ideas for Chevalier to find in his Testamentary Letter. Though Galois lamented in one of his last letters that fate had not given him enough life for the country to know his name, he ultimately managed to achieve his wish through the profundity of his work.

Galois deserves our sympathy. The young boy lived immersed in a period of political turmoil, and he even lost his father to political intrigue. As for his mathematical endeavors, the loss of Galois's submission to the *Grand Prix de Mathématiques* seems inexcusable. Furthermore, most young mathematicians had to lead an arduous, thankless life (as they do now) in an effort to climb the scientific hierarchy. The similarity of Galois's struggles to their own helps explain why he has inspired so many in their beginning years. I have argued that Cauchy and Poisson were not nearly so disdainful to Galois as legend has made them out to be. Nevertheless, Galois's story shows the great potential that lies in certain budding mathematicians and that the old gatekeepers of mathematics would do well to cultivate.

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